



جامعة الأمير سطام بن عبد العزيز
Prince Sattam Bin Abdulaziz University

General Physics (1)

PHYS 1010



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Chapter (1)

Units and Dimensions



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Chapter (1)

Units and Dimensions

Content

Fundamental and derivatives units and measurement system

Learning Objectives

1. The base quantities in the System International.
2. Name the most frequently used prefixes for SI units.
3. Change units (here for length, area, and volume) .
4. Defined by relationships to base quantities
5. Each quantities defined by a standard, and given a unit
6. Verify the correction of the equations.
7. Explain how to obtain the units of the physical constants
8. Explain how to obtain the equation represents a special physical law

1.1. What is Physics?

Physics is a natural science concerned primarily with the principles and laws governing the behavior of the inanimate world around us . As a science it involves many different subjects . These subjects may be divided and grouped under one of two headings , classical physics and modern physics . Classical physics is concerned largely with macroscopic bodies, that is , with those phenomena in which the objects involved are large and can be seen with the eye . Modern physics, on the other hand, is concerned primarily with the submicroscopic world, that is, with those phenomena in which the structure and the behavior of individual atoms and molecules are of prime importance.

1.2. Physical Quantities

Physical quantities are often divided into:

- a- **Fundamental (Principal) quantities**, usually are length, mass and time.
- b- **Derived quantities**, are those whose defining operations based on other physical quantities. Examples of quantities usually viewed as derived are velocity, acceleration, density and volume.

1.2.1. Standard of Length

The first truly international standard of length (in 1960) was a bar of platinum – iridium alloy called the standard **meter**, kept at the International Bureau of Weights and Measures near Paris, France. The distance between two fine lines engraved on gold plugs near the ends of platinum- iridium bar at 0 °C was defined to be **one meter**.

In the 1960's and 1970's the meter was defined as 1 650 763.73 wavelengths of orange –red light emitted from a krypton-86 lamp. However, in 1983, the meter (m) was redefined as the distance traveled by light in vacuum during a time of $(\frac{1}{299\,792\,457})$ seconds.

1.2.2. Standard of Mass

Because platinum – iridium is a stable alloy, the SI unit of mass is the kilogram (kg), is defined as the mass of a specific platinum- iridium alloy cylinder kept at the International Bureau of Weights and Measures at Sèvres, France. This mass standard was established in 1887 and has not been changed since that time.

1.2.3. Standard of Time

Before 1960, the standard of time was defined in terms of the mean solar day for the year 1900. (A solar day is the time interval between successive appearances of the Sun at the highest point it reaches in the sky each day). The second was defined as $(\frac{1}{60})(\frac{1}{60})(\frac{1}{24})$ of a mean solar day.

In 1967, the second was redefined to take advantage of the high precision attainable in a device known as an atomic clock, which uses the characteristic frequency of the cesium-133 atom as the "reference clock." The second is now defined as 9,192,631,770 times the period of vibration of radiation from the cesium atom.

1.2.4. Principal Quantities

1) Length 2) Mass 3) Time 4) Temperature 5) Electric Current 6) Luminous Intensity
7) Amount of Substance

1.2.5. Write the Physical Quantity

- A physical quantity is always the product of two quantities, a number and a unit; for example $(T = 300.15K)$, $(W = 20N)$, $(R = 8.3145 J.K^{-1}.mol^{-1})$ or $(V = 15m/s)$.
- In applications of mathematics in the sciences, numbers by themselves have no meaning unless the units of the physical quantities are specified. It is important to know what these units are, but the mathematics does not depend on them.
- Units obey the laws of ordinary algebra, and can be manipulated like numbers.
- The units associated with a physical quantity depend on the dimensions of that quantity. There are seven physical quantities that are described as being dimensionally independent (The Basic Quantities).

1.3. Systems of Units

Three different systems of units are most commonly used in science and engineering. They are:

- 1-the meter-kilogram-second or mks system.
- 2-the Gaussian system, in which the fundamental mechanical units are the centimeter, the gram, and the second (a cgs system).
- 3-the British engineering system (a foot- pound- second or fps system). Table 1.1 lists the three different systems of units.

Quantity	Units systems		
	SI System OR (MKS)	Gaussian OR (CGS)	British System
Length	m	cm	ft
Time	s	s	s
Mass	Kg	gm	Ib (Unit of force)

Table (1.1) lists the three different systems of units

1.3.1.The International System of Units

Physical Quantity	Name of Unit	Symbol
Length	meter	m
Mass	kilogram	kg
Time	second	s
Electric Current	Ampere	A
Thermodynamic Temperature	Kelvin	K
Amount of Substance	mole	mol
Luminous Intensity	candela	Cd

Table (1.2) Physical Quantity in SI system

Notes: Write the unit always small letters but if derived from the name of the world write in Capital letters.

Some relation between units

$$1mm = 10^{-3}m \quad \& 1km = 10^3m \quad \& 1\mu m = 10^{-6}m$$

$$1nm = 10^{-9}m \quad \& 1Mm = 10^6m \quad \& 1Gm = 10^9m$$

$$1yd = 3ft \quad \& 1ft = 12in \quad \& 1in = 2.54cm$$

$$1m = 3.281ft \quad \& 1mile = 1609m$$

$$1liter = 10^3cm^3 \quad \& 1gallon = 3.79liter$$

$$1Ib = 4.448N \quad \& 1N/m^2 = 1.451 \times 10^{-4}Ib/in^2$$

1.4 Dimensional Analysis

The word dimension has a special meaning in physics. It denotes the physical nature of a quantity. Whether a distance is measured in units of feet or meters or fathoms, it is still a distance. We say its dimension is length. The symbols used to specify the dimensions of length, mass, and time are L, M, and T. We shall often use brackets [] to denote the dimensions of a physical quantity.

In summary; dimensions: In order to add or subtract two quantities, they must have the same dimensions. Table 1.3 lists the dimension of basic and derived Quantities.

	The QUANTITIES	DERIVED UNITS	The Dimension
Basic Quantities	LENGTH	m	[L]
	TIME	S	[T]
	MASS	kg	[M]
Derived Quantities	AREA	m²	[L²]
	VOLUME	m³	[L³]
	DYNAMIC VISCOSITY	kg.m⁻¹.S⁻¹	[M L⁻¹T⁻¹]
	VELOCITY	m/s	[LT⁻¹]
	ACCELERATION	m/s²	[LT⁻²]
	FORCE	kg.m.s⁻²	[M LT⁻²]
	FREQUENCY	S⁻¹	[T⁻¹]
	DENSITY	Kg/m³	[M L⁻³]
	ELASTIC MODULUS	kg.m⁻¹.s⁻²	[M L⁻¹T⁻²]
	WORK(Energy)	kg m².s⁻²	[M L²T⁻²]
	ENERGY POTENTIAL	kg m².s⁻²	[M L²T⁻²]
	ENERGY KINETIC	kg m².s⁻²	[M L²T⁻²]
	POWER	kg m².s⁻³	[M L²T⁻³]
	PRESSURE	kg.m⁻¹.s⁻²	[M L⁻¹T⁻²]

Table (1.3) lists the dimension of basic and derived Quantities.

1.4.1 Uses of Dimensional Analysis

- (1)- To verify the correction of the equations.
- (2)- To obtain the units of the physical constants
- (3)- To obtain the equation represents a special physical law

Example (1.1)

Show that the following equation is dimensionally correct? $X = V_0 t + \frac{1}{2} a t^2$

Where x distance ,V: velocity, a: acceleration ,t: time

Solution

The dimensional form of the equation ($X = V_0 t + \frac{1}{2} a t^2$) is

$$LHS = [L]$$

$$RHS = [LT^{-1}] \times [T] + [LT^{-2}] \times [T]^2 = [L] \quad , \text{THEN}$$

RHS = LHS $\Rightarrow$ The equation is dimensionally correct.

Example (1.2)

Check the validity of this expression $X = \frac{1}{2} a t^2$.

Where X distance , a: acceleration ,t: time

Solution

The dimensional form of the equation ($X = \frac{1}{2} a t^2$) is

$$LHS = [L]$$

$$RHS = [LT^{-2}] \times [T]^2 = [L]$$

The equation is dimensionally correct.

Example (1.3)

Check the validity of this expression $V = V_0 + at$

Where V: velocity, a: acceleration ,t: time

Solution

$$LHS = [LT^{-1}]$$

$$RHS = [LT^{-1}] + [LT^{-2}][T] = [LT^{-1}]$$

The equation is dimensionally correct

Example (1.4)

We have $a = k r^n V^m$ where a acceleration of a particle ,uniform speed v in a circle of radius r , k is a constant , Determine the values of n and m .

Solution

$$a = k r^n V^m$$

L.H.S. = Dimensions of R.H.S

$$LHS = [LT^{-2}]$$

$$[L][T^{-2}] = [L]^n [LT^{-1}]^m = [L]^{n+m} [T]^{-m}$$

$$\text{For } [T]; \quad -m = -2 \quad \rightarrow \quad m = 2$$

$$\text{For } [L]; \quad n + m = 1 \quad \rightarrow \quad n = 1 - m = 1 - 2 = -1$$

Example (1.5)

The period (T) of a simple pendulum is the time for one complete swing. How does (T) depends on the mass (m) of the bob, the length (L) of the string and gravitational (g)?

Solution

Suppose that the required law is given by

$$T = (\text{constant}) m^a L^b g^c$$

$$[M]^0 [L]^0 [T] = [M]^a [L]^b [LT^{-2}]^c = [M]^a [L]^{b+c} [T]^{-2c}$$

$$\text{For } [M]; \quad a = 0$$

$$\text{For } [T]; \quad 1 = -2c \quad \rightarrow \quad c = -\frac{1}{2}$$

$$\text{For } [L]; \quad 0 = b + c \quad \rightarrow \quad b = -c \quad \rightarrow \quad b = \frac{1}{2}$$

$$T = (\text{constant}) L^{\frac{1}{2}} g^{-\frac{1}{2}} = (\text{constant}) \sqrt{\frac{L}{g}}$$

Example(1.9)

The speed of a particle varies in time according to $V = At - Bt^3$. What are the dimensions of A and B ?

Solution

$$V = A t - B t^3 \rightarrow [LT^{-1}] = [A][T] - [B][T]^3$$

$$[L][T^{-1}] = [A][T] - [B][T]^3$$

$$\text{So, } [L][T^{-1}] = [A][T], \quad [A] = [LT^{-1}]$$

$$\text{And, } [L][T^{-1}] = [B][T]^3, \quad [B] = [LT^{-3}]$$

Example (1.10)

According to Newton's law of gravitation there is an attractive force between particles given by: $F = G \frac{m_1 m_2}{r^2}$, where: F the force, r is the distance between the particles, m_1, m_2 masses, What are the dimensions of G and the unit of G in SI?

Solution

$$F = G \frac{m_1 m_2}{r^2}$$

Substituting the dimensions of various quantities, we have

$$[M L T^{-2}] = [G] [M] [M] / [L^2] \rightarrow [M L T^{-2}] = [G] [M^2] [L^{-2}]$$

$$[G] = [L^3] [M^{-1}] [T^{-2}]$$

Also; The unit of G is $\text{m}^3/\text{Kg.s}^2$

Example(1.11)

Einstein relation between the energy, E, of a body and its mass is given by : $E = mc^2$, where c is the free space speed of light . Check the correctness of this equation.

Solution

$$\text{LHS : } [E] = [M L^2 T^{-2}]$$

$$\text{RHS : } [mc^2] = [M L^2 T^{-2}]$$

i.e. $[LHS] = [RHS]$, therefore the equation is correct .

1.5. Standard Form

Standard form or scientific notation is used to express magnitude in a simpler way.

In scientific notation, a numerical magnitude can be written as: $A \times 10^n$, where $1 \leq A < 10$ and n is an integer.

For each of the following, express the magnitude using a scientific notation.

1-20000000

2-345000

3-0.0000023

4-0.00000006

5-123402123100

Solution :

1- 2×10^7

2- 3.45×10^5

3- 2.3×10^{-6}

4- 6×10^{-8}

5- 1.2×10^{11}

1.6. Prefixes, cont.

Prefixes correspond to powers of 10.

Each prefix has a specific name and a specific abbreviation.

The prefixes can be used with any basic units.

They are multipliers of the basic unit.

Examples:

- $1 \text{ Km} = 10^3 \text{ m}$
- $1 \text{ }\mu\text{g} = 10^{-6} \text{ g}$

Nome	Symbol	Factor	Nome	Symbol	Factor
yotta	Y	10^{24}	deci	d	10^{-1}
zetta	Z	10^{21}	centi	c	10^{-2}
exa	E	10^{18}	milli	m	10^{-3}
peta	P	10^{15}	micro	μ	10^{-6}
tera	T	10^{12}	nano	n	10^{-9}
giga	G	10^9	pico	p	10^{-12}
mega	M	10^6	femto	f	10^{-15}
kilo	K	10^3	atto	a	10^{-18}
hecto	H	10^2	zepto	z	10^{-21}
deca	Da	10^1	yocto	y	10^{-24}

1.7. Conversion of Units

When units are not consistent, you may need to convert to appropriate ones.

Units can be treated like algebraic quantities that can cancel each other out.

Chain-link conversion

$$1\text{ day}=24\text{ h}$$

$$1\text{ h}=60\text{ min}$$

$$1\text{ min}=60\text{ s}$$

$$1\text{ mile}=1609\text{ m}$$

$$1\text{ km}=1000\text{ m}$$

$$1\text{ cal}=4.18\text{ J}$$

$$1\text{ year}=365\text{ days}$$

$$\begin{aligned}\text{Conversion factor: } (1\text{ min}/60\text{ s}) &=1 \\ (60\text{ s}/1\text{ min}) &=1\end{aligned}$$

Converting feet to meters:

$$1\text{ ft} = 30.48\text{ cm} = 0.3048\text{ m} \quad (\text{this is a conversion factor})$$

$$\text{Or: } 1 = (0.3048\text{ m} / 1\text{ ft})$$

Then:

$$320\text{ ft} \times (0.3048\text{ m} / 1\text{ ft}) = 97.54\text{ m}$$

Note that the units cancel properly – this is the key to using the conversion factor correctly!

1.8.Problems**Choose The Correct Answer in Each of The Followings:**

1) $(5.0 \times 10^4) \times (3.0 \times 10^{-6}) =$

- A) 1.5×10^{-3}
- B) 1.5×10^{-1}
- C) 1.5×10^1
- D) 1.5×10^3

2) $5.0 \times 10^5 + 3.0 \times 10^6 =$

- A) 8.0×10^5
- B) 8.0×10^6
- C) 5.3×10^5
- D) 3.5×10^6

3) The SI base units have the dimensions of:

- A) mass, weight, work
- B) energy, density, time
- C) mass, length, time
- D) weight, power, time

4) The SI base unit for mass is:

- A) gram
- B) pound
- C) kilogram
- D) ounce

5) A gram is:

- A) 10^{-6} kg
- B) 10^{-3} kg
- C) 1 kg
- D) 10^3 kg

6) 1 mil is equivalent to 1609 m so the speed of 85 mil per hour equals (in m/s) :

- A) 15
- B) 38
- C) 85
- D) 190

7) 1 ft is equivalent to 0.3048 m. A cube with an edge of 1.5 ft has a volume (m^3) of:

- A) 1.29×10^2
- B) 9.88×10^{-2}
- C) 10.55
- D) 9.56×10^{-2}

8) A sphere with a radius of 1.7 cm has a volume of in m^3 :

- A) 2.06×10^{-5}
- B) 9.10×10^{-4}
- C) 3.66×10^{-3}
- D) 0.11

9) A sphere with a radius of 1.7 cm has a surface area of (in m^2)

- A) 2.13×10^{-5}
- B) 9.11×10^{-4}
- C) 3.63×10^{-3}
- D) 0.11 m^2

10) A square with an edge of exactly 2 cm has an area of (m^2):

- A) 4×10^{-6}
- B) 4×10^{-4}
- C) 2×10^{-2}
- D) 4×10^2

11) A nanosecond is:

- A) 10^9 s
- B) 10^{-9} s
- C) 10^{-6} s
- D) 10^{-15} s

12) During a short interval of time the speed v in m/s of an automobile is given by $v = at^2 + bt^3$, where the time t is in seconds. The units of a and b are respectively:

- A) $\text{m} \cdot \text{s}^2$; $\text{m} \cdot \text{s}^4$
- B) s^3/m ; s^4/m
- C) m/s^2 ; m/s^3
- D) m/s^3 ; m/s^4

13) A sphere has a radius of 21 cm and a mass of 1.9 kg. Its mass density is about:

- A) $4.9 \times 10^{-5} \text{ kg}/\text{m}^3$
- B) $49 \text{ kg}/\text{m}^3$
- C) $2.0 \times 10^{-2} \text{ kg}/\text{m}^3$
- D) $16 \text{ kg}/\text{m}^3$

14) The dimension of force is

- A) $[\text{M L T}]$
- B) $[\text{M L}^{-1} \text{T}]$
- C) $[\text{M L T}^{-2}]$
- D) $[\text{M L}^{-1} \text{T}^{-1}]$

15) A quantity has a dimension of $[M L^2 T^{-2}]$ then its unit is

- A) $m \cdot Kg^2 \cdot sec^{-2}$
- B) $Kg \cdot m^{-2} \cdot sec^2$
- C) $Kg^2 \cdot m \cdot sec^2$
- D) $Kg \cdot m^2 \cdot sec^{-2}$

16) A cubic box with an edge of exactly 3 cm has a volume of:

- A) $27 \times 10^{-9} m^3$
- B) $27 \times 10^{-6} m^3$
- C) $9 \times 10^{-3} m^3$
- D) $9 \times 10^3 m^3$

17) Six million seconds is approximately:

- A) One day
- B) Ten days
- C) Two months
- D) One year

18) Suppose $A = B^n C^m$, where A has dimensions LT , B has dimensions $L^2 T^{-1}$, and C has dimensions LT^2 . Then the exponent's n and m have the values:

- A) $2/3; 1/3$
- B) $2; 3$
- C) $4/5; -1/5$
- D) $1/5; 3/5$

19) The prefix centi equals

- A) 10^{-10}
- B) 10^{-2}
- C) 10^{-1}
- D) 10^{-9}

20) A quantity has a unit of $\frac{Kg^2}{m \cdot sec^2}$ then its dimension is

- A) $[M^2 L T]$
- B) $[M^2 L^{-1} T]$
- C) $[M L T^{-2}]$
- D) $[M^2 L^{-1} T^{-2}]$

21) The prefix micro equals

- A) 10^{-3}
- B) 10^{-6}
- C) 10^{-1}
- D) 10^{-9}

22) The prefix Mega equals

- A) 10^{-3}
- B) 10^{-6}
- C) 10^6
- D) 10^{-9}

23) The dimension of velocity is

- A) $[L T^2]$
- B) $[L^{-1} T^{-2}]$
- C) $[M T^{-1}]$
- D) $[L T^{-1}]$

24) The dimension of acceleration is

- A) $[L^2 T^{-1}]$
- B) $[L^{-1} T]$
- C) $[L T^{-2}]$
- D) $[M L^{-1} T^{-2}]$

25) If length of pendulum is increased by 2%. The time period will ($T = 2\pi\sqrt{\frac{L}{g}}$)

- A) Increases by 1%
- B) Decreases by 1%
- C) Increases by 2%
- D) Decreases by 2%

26) The prefix milli equals

- A) 10^{-3}
- B) 10^{-6}
- C) 10^{-1}
- D) 10^{-9}

27) The prefix Tera equals

- A) 10^{-3}
- B) 10^{12}
- C) 10^{-1}
- D) 10^{-9}

28) The dimension of density is

- A) $[L T]$
- B) $[L^{-1} T]$
- C) $[M L^{-3}]$
- D) $[L T^{-3}]$

29) The dimension of volume is

- A) [L T]
- B) [L⁻¹ T]
- C) [L³]
- D) [L⁻³]

30) The dimensional formula of coefficient of viscosity η is (if $F = 6\pi\eta r.v$)

Where v : velocity, r : distance , F : force

- A) [M L T⁻¹]
- B) [M⁻¹L²T⁻²]
- C) [M L⁻¹T⁻¹]
- D) [L⁻¹ T]

31) The dimension of area is

- A) [L T]
- B) [L⁻¹ T]
- C) [L²]
- D) [L⁻²]

32) The dimension of frequency is

- A) [L T]
- B) [T⁻¹]
- C) [T²]
- D) [T⁻²]

33) Assume the equation $X = At^3 + Bt$ describes the motion of a particular object, with x the length and t the time. Determine the dimensions of the constants A and B .

- A) [L/T³, L/T]
- B) [M⁻¹L²T⁻²]
- C) [L/T², L/T⁴]
- D) [L⁻¹ T, T²]

34) Someone is (60 in) tall. how tall is the person in centimeters?

- A) 23.6
- B) 9.7
- C) 152.4
- D) 16.764

35) The standard kilogram is a platinum–iridium cylinder 39.0 mm in height and 39.0 mm in diameter, See figure (1.1). What is the density of the material in (Kg/m^3) :

- A) $\rho = 214$
- B) $\rho = 21464.27$
- C) $\rho = 21.4$
- D) $\rho = 500$

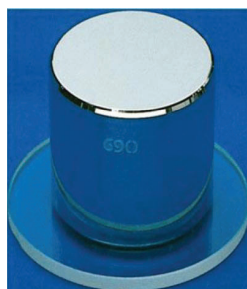


Figure (1.1)

36) A solid piece of lead has a mass of 23.94 g and a volume of 2.10 cm^3 . From these data, calculate the density of lead in SI units (kilograms per cubic meter).

- A) 1.140×10^4
- B) 1.140×10^{-2}
- C) 11.40
- D) 500

37) A rectangular building lot has a width of 75 ft and a length of 125 ft. Determine the area of this lot in square meters.

- A) 870.97
- B) 87.088
- C) 9375
- D) 200

Q2: Which of the following equations are dimensionally correct?

- (a) $V_f = V_i + aX$ (b) $y = (2 \text{ m}) \cos(KX)$, where $K = 2 \text{ m}^{-1}$

where V : velocity , a :acceleration and X : distance

Solution:

$$V_f = V_i + aX$$

$$[L.T^{-1}] = [L.T^{-1}] + [L.T^{-2}].[L] = [L.T^{-1}] + [L^2.T^{-2}]$$

The equation is not dimensionally correct.

$$y = (2 \text{ m}) \cos(KX), \text{ where } K = 2 \text{ m}^{-1}$$

$$[L] = [L] \cos(2L^{-1}.L)$$

$$[L] = [L] \cos(2) = [L]$$

The equation is dimensionally correct.

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Chapter (2)

Elasticity



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Chapter (2)

Elasticity

Content

Elastic and Plastic materials – Stress – Strain – Thermal stress - Stress-Strain Curve – Young modulus – Shear modulus – Bulk modulus – Strain energy.

Learning Objectives

1. Understand the principle concepts of the elasticity
2. For tension and compression, apply the equation that relates stress to strain and Young's modulus.
3. Distinguish between yield strength and ultimate strength.
4. For shearing, apply the equation that relates stress to strain and the shear modulus.
5. For hydraulic stress, apply the equation that relates fluid pressure to strain and the bulk modulus.

2.1: Mechanical Properties of Metals

Many materials may be deformed when external forces exert on them.

- **Elasticity materials:** If the material restore to its original shape and size after removing the load from it, it's said to be elastic.
- **Plastic materials:**
If the material fails to restore its original dimensions after removing the applied stress, it's said to plastic.
- **Elastic Modulus:** Is the constant of each matter and equal ratio between stress and strain
- **Concepts of Stress and Strain**

2.1.1 Stress (σ)

It is the instantaneous perpendicular force (F) per unit area (A).

i.e. is related to the force causing the deformation

$$\sigma = \frac{F}{A} \quad \text{N/m}^2 \quad \text{or} \quad \text{lb/in}^2 \quad (2.1)$$

2.1.2 Strain (ε)

Is a measure of the degree of deformation?

2.1.3 Stress –Strain Diagram

Let us consider a typical **stress –strain diagram** for a ductile metal as shown in

Figure (2.1)

The stress and strain are proportional until point a is reached. The point **a** is called the **proportional limit** of the material.

From **a** to **b**, stress and strain are not proportional, but nevertheless, if the load is removed at any point between **o** and **b**, the curve will retraced and the material will

return to its original state. In the region **ob**, the material is said to be **elastic** or exhibit **elastic behavior** and, the point **b** is called the elastic **limit**, or the yield point.

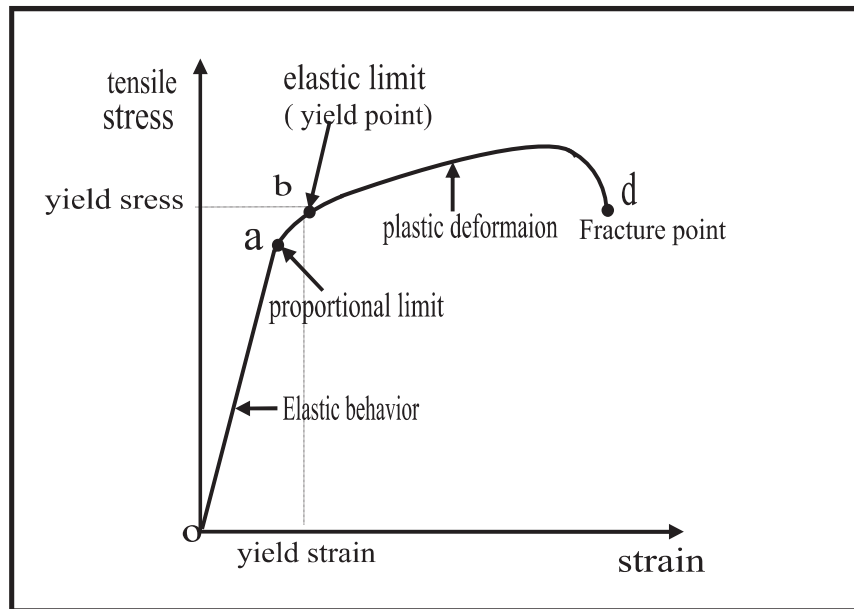


Figure (2.1) **stress –strain diagram**

Up to point **b** (**elastic limit or yield point**), the forces exerted by the material are conservative, when the material returns to its original shape, work done in producing the deformation is recovered. The deformation is said to be reversible. (the **yield point** is a point beyond proportional limit. At this point the removal of the loading results in permanent strain)

Further increase of the load beyond **b** produces a large increase in the strain (even if the stress decreases) until a point **d** is reached at which fracture takes place. From **b** to **d**, the material is said to undergo **plastic deformation**. A plastic deformation is **irreversible**.

Stress required to cause actual **fracture** of a material is called the **breaking stress** or the **ultimate strength**. If large plastic deformation takes place between the elastic limit (**b**) and the fracture point (**d**), the metal is said to be **ductile**. If, however, fracture occurs soon after the elastic limit (**b**) is passed, the metal is said to be **brittle**.

Safety Factor:

- For all engineering materials it is not allowed to apply stress on any material beyond its elastic limit.
- Even within the elastic region the stress must be smaller than the proportional limits.

- Also a factor of safety is used by all international standards to keep the material safe to use . So that the allowed stress; is a fraction of the stress at the proportional limit.

$$\text{Safety factor} = \frac{\text{Stress at the proportional limit}}{\text{Allowed stress}} \quad (2.2)$$

2.1.4 Hooke's Law

“The Stress is directly proportional to the Strain”

$$\sigma = Y\varepsilon$$

Y: is the Elastic modulus or Young's modulus

The greater the modulus, the stiffer the material, or, the smaller the strain results from the application of the stress. Figure (2.2)

- If a rod is stretched by a force F_1 distance ΔL , then

$$Y = \frac{F_1 L}{A \Delta L} \Rightarrow F = \frac{YA}{L} \Delta L \quad (2.3)$$

Since Y, A and L is constant for each material,

Then we can write $K = \frac{YA}{L}$ (Constant Force)

So, we can write Equation (2.1) as $F_1 = K \Delta L$ (2.4)

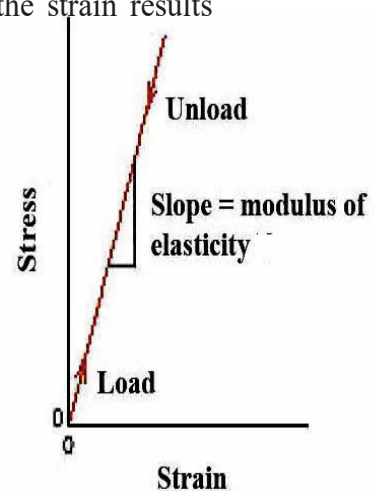


Figure (2.2)

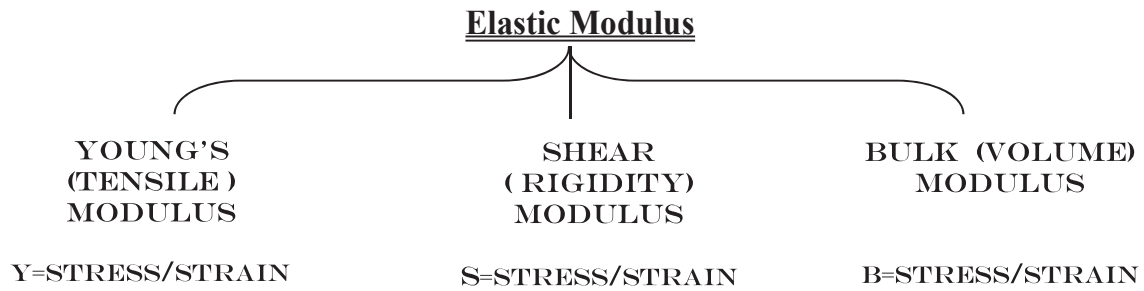
Note That;

$$1 \text{ lb/in}^2 (\text{psi}) = 6891 \text{ N/m}^2 (\text{Pa}) \quad 1 \text{ N/m}^2 (\text{Pa}) = 1.451 \times 10^{-4} \text{ lb/in}^2 (\text{psi})$$

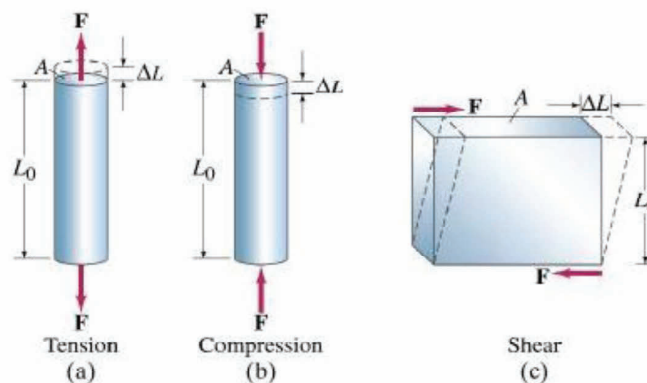
2.2 Elastic Modulus

The stress required to produce a given strain depends on the nature of the material under stress. The ratio of stress to strain, or the stress per unit strain, is called an elastic modulus of the material. Corresponding to the three types of strains (tensile, shear, and volume strain) we have three elastic modulus, Figure (2.3):

- Young's modulus of elasticity; Y : It corresponds to tensile strain .
- Shear modulus (or modulus of rigidity); S : It corresponds to shearing strain.
- Bulk modulus (or volume modulus); B: It corresponds to volumetric strain .

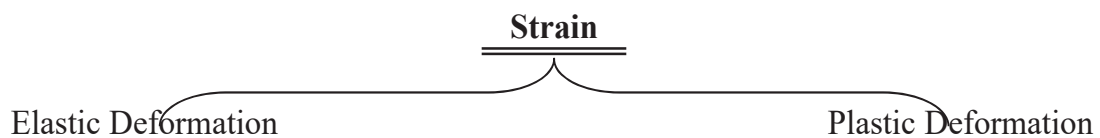


Stress Different Types



The response to different types of stress can differ greatly for the same material.

Figure (2.3)



2.2.1. Young's Modulus: (Y) It is the ratio between the stress and the strain ,See Figure (2.4).

$$Y = \frac{F / A}{\Delta L / L} = \frac{\sigma}{\epsilon} \quad \text{N/m}^2 \quad \text{or} \quad \text{lb/in}^2 \quad (2.5)$$

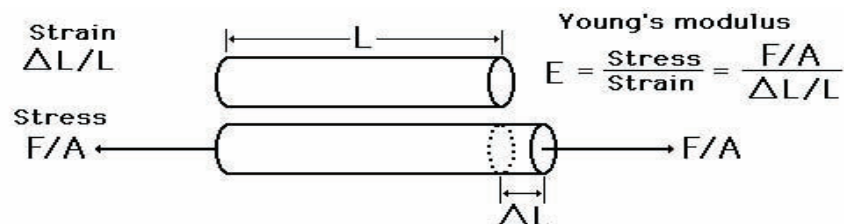


Figure (2.4)

Where:

tensile Stress (σ)

It is the instantaneous perpendicular force (F) per unit area (A).

$$\sigma = \frac{F}{A} \quad \text{N/m}^2 \quad \text{or} \quad \text{lb/in}^2 \quad (2.6)$$

tensile Strain (ε)

It is the ratio between the change in length (ΔL) and the original length (L_0).

$$\varepsilon = \frac{\Delta L}{L_0} \quad (2.7) \quad , \quad \Delta L : \text{is the elongation or stretch}$$

Example (2.1)

A 80 Kg mass is hung on a steel wire having 18 m long and 3 mm diameter. What is the elongation of the wire, If Young's modulus for steel is $21 \times 10^{10} \text{ N/m}^2$?

Answer

$$\begin{aligned} \text{We have } Y &= \frac{F/A}{\Delta L / L_0} \quad \text{so the elongation is } \Delta L = \frac{F \cdot L_0}{A \cdot Y} \\ \Delta L &= \frac{80 \times 9.8}{\pi(0.0015)^2} \times \frac{18}{21 \times 10^{10}} = 0.0095 \text{ m} = 9.5 \text{ mm} \end{aligned}$$

Example (2.2)

A piece of copper originally 0.305m long is pulled in tension with a stress of $276 \times 10^6 \text{ Pa}$. If the deformation is entirely elastic, what will be the resultant elongation?
 $Y_{\text{cu}} = 11 \times 10^{11} \text{ N/m}^2$

Answer

$$\begin{aligned} \text{We have } Y &= \frac{F/A}{\Delta L / L_0} \quad \text{then:} \\ \Delta L &= \frac{F \cdot L_0}{A \cdot Y} = \frac{\sigma L_0}{Y} = \frac{(267 \times 10^6)(0.305)}{11 \times 10^{11}} = 0.76 \times 10^{-3} \text{ m} \end{aligned}$$

Example (2.3)

A telephone wire 120 m long and 2.2 mm in diameter is stretched by a force of 380 N. What is the longitudinal stress? If the length after stretching is 120.10 m, what is the longitudinal strain? Determine Young's modulus for the wire?

Answer

$$\begin{aligned} A &= \pi r^2 = \pi(1.1 \times 10^{-3})^2 = 3.8 \times 10^{-6} \text{ m}^2 \\ \sigma &= \frac{F}{A} = \frac{380}{3.8 \times 10^{-6}} = 10^8 \text{ N/m}^2 = 100 \text{ MPa} \\ \Delta L &= 120.1 - 120.1 \text{ m} \end{aligned}$$

$$\varepsilon = \frac{\Delta L}{L_0} = \frac{0.1}{120} = 8.33 \times 10^{-4}$$

$$Y = \frac{\sigma}{\varepsilon} = \frac{10^8 \text{ N/m}^2}{8.33 \times 10^{-4}} = 12 \times 10^{10} \text{ N/m}^2 = 120 \text{ GPa}$$

Example (2.4)

A structure steel rod has a radius 9.5 mm and a length 81 cm.

A force F of $6.2 \times 10^4 \text{ N}$ stretches it axially. ($Y_{\text{steel}} = 2 \times 10^{11} \text{ N/m}^2$)

- Calculate the stress in the rod?
- Calculate the elongation of the rod under this load?
- Calculate the strain?

Answer

$$\text{Tensile Stress } (\sigma) = \frac{F}{A} = \frac{F}{\pi r^2} = \frac{6.2 \times 10^4}{\pi (9.5 \times 10^{-3})^2} = 2.19 \times 10^8 \text{ N/m}^2$$

$$\text{Tensile Strain } \varepsilon = \frac{\Delta L}{L_0}, Y = \frac{\sigma}{\varepsilon}$$

$$\Delta L = \varepsilon \times L_0 = \frac{\sigma}{Y} \times L_0 = \frac{2.19 \times 10^8}{2 \times 10^{11}} \times 81 \times 10^2 = 8.87 \times 10^{-4} \text{ m}$$

$$\varepsilon = \frac{\sigma}{Y} = \frac{2.19 \times 10^8}{2 \times 10^{11}} = 1.1 \times 10^{-3}$$

Example (2.5)

A certain wire stretches 0.90 cm when outward forces with magnitude F are applied to each end. The same forces are applied to a wire of the same material but with three times the diameter and three times the length. The second wire stretches:

Answer

$$\text{The Young's modulus for the wire } Y = \frac{F \times L}{\Delta L \times A}$$

$$\text{The Young's modulus for the wire (1) } Y = \frac{F \times L_1}{\Delta L_1 \times A_1} \quad (1)$$

$$\text{The Young's modulus for the wire (2) } Y = \frac{F \times L_2}{\Delta L_2 \times A_2} \quad (2)$$

The equation (1) equals the equation (2)

$$\frac{F \times L_1}{\Delta L_1 \times A_1} = \frac{F \times L_2}{\Delta L_2 \times A_2} \Rightarrow$$

$$\Delta L_2 = \frac{\Delta L_1 \times A_1 \times L_2}{A_2 \times L_1}$$

$$\Delta L_2 = \frac{0.9 \times (\pi \times r_1^2) \times 3L_1}{(\pi \times (3r_1)^2 \times L_1)} = \frac{0.9 \times \pi \times r_1^2 \times 3L_1}{\pi \times 9r_1^2 \times L_1} = 0.3 \text{ cm}$$

2.2.2. Shear Modulus (Elasticity in Shape) (S)

$$\text{Shear Modulus (S)} = \frac{\text{Shear Stress}}{\text{Shear Strain}} = (F/A)/\theta = (F/A)/(\Delta x/h) \quad \text{N/m}^2 \quad \text{or} \quad \text{lb/in}^2$$

Where

$$\text{Shear Stress} = \frac{F_t}{A} \quad \text{N/m}^2 \quad \text{or} \quad \text{lb/in}^2$$

Shear modulus: Elasticity of shape

When an object is subjected to a force parallel to one of its faces while the opposite face is held fixed by another force, the stress in this case is called a **shear stress**, see Figure (2.5) . The shear stress is defined as the ratio of the tangential force F to the area A of the face being sheared . The shear strain is defined as the ratio of the horizontal distance x that the sheared face moves, to the height of the object h (assuming that, for small distortions, no change in volume occurs with this deformation) . Thus the shear modulus is :

$$S = \text{shear stress/shear strain}$$

$$\text{Shear stress} = F/A$$

$$\text{Shear strain} = \Delta x/h = \tan \theta \cong \theta \quad (\text{if } x \ll h)$$

$$S = \frac{F}{\frac{A}{\frac{\Delta x}{h}}} \quad \text{or} \quad S = \frac{F}{A} \cdot \frac{h}{\Delta x} = \frac{F}{A} \cdot \frac{1}{\theta} \quad (2.8)$$

The shear modulus (or modulus of rigidity) , S has a significance for solid materials only. The SI units of S are that of stress, i.e. N/m^2 .

When a material is subjected to shear stress the volume will not change.

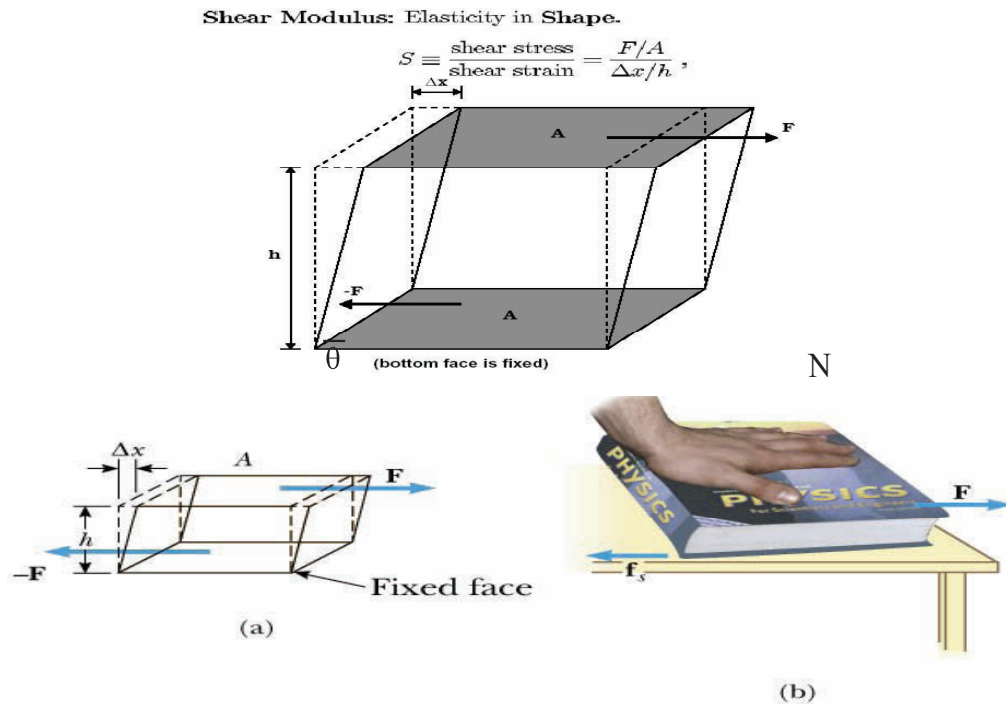


Figure (2.5)

Note : Only Solids have shear and Young's Moduli(Liquid can flow).

Example (2.6)

A horizontal force of 1.2 N is applied to the top of a stack of pancakes 13 cm in diameter and 9 cm high. The result is a 2.5 cm shear. Find the shear modulus.

Answer

$$\text{Shear Modulus}(S) = \left(\frac{F}{A}\right) / \theta = (F/A) / (\Delta x/h)$$

$$S = \frac{F h}{A \Delta x} = \frac{F h}{\pi r^2 \Delta x} = \frac{1.2 \times 0.09}{\pi (0.13/2)^2 \times (0.025)} = 3.255 \times 10^2 \text{ N/m}^2$$

Example (2.7)

A tangential force of 1000 N exerted upon the upper surface of a cube of 10 cm edge, if the result displacement was 0.1 cm then the Shear Modulus of the cube material equals

$$S = \frac{F}{\frac{A}{h} \Delta x} \Rightarrow \text{or } S = \frac{F}{A} \cdot \frac{h}{\Delta x} \Rightarrow$$

$$S = \frac{1000}{(10 \times 10^{-2})^2} \frac{10 \times 10^{-2}}{0.1 \times 10^{-2}} = 10^7 \text{ N/m}^2$$

2.2.3. Bulk Modulus: Volume Elasticity

Bulk modulus is defined as the negative ratio of volume stress to the volume strain . When a force is applied normally to the surface of a body and a change in volume takes place, the strain is known as volume strain . It is measured by the change in volume per unit volume, that is:

$$\text{Volume strain} = \Delta V / V$$

Where Δv is the change in volume produced by the force F in the original volume V
By definition the bulk modulus of elasticity is given by :

$$B = \text{volume stress} / \text{volume strain}$$

$$B = - \frac{\frac{F}{A}}{\frac{\Delta V}{V}} \rightarrow B = - \frac{\Delta P \times V}{\Delta V} \quad (2.9)$$

The minus sign is included in the definition of B because an increase in applied pressure causes a decrease in volume (negative ΔV) and vice versa. The SI units of B are the same as those of pressure, i.e. N/m^2 .

Figure (2.6) shows that when a cube of solid is undergoes a change in volume but no change in shape , the cube is compressed on all sides by forces normal to its six faces.

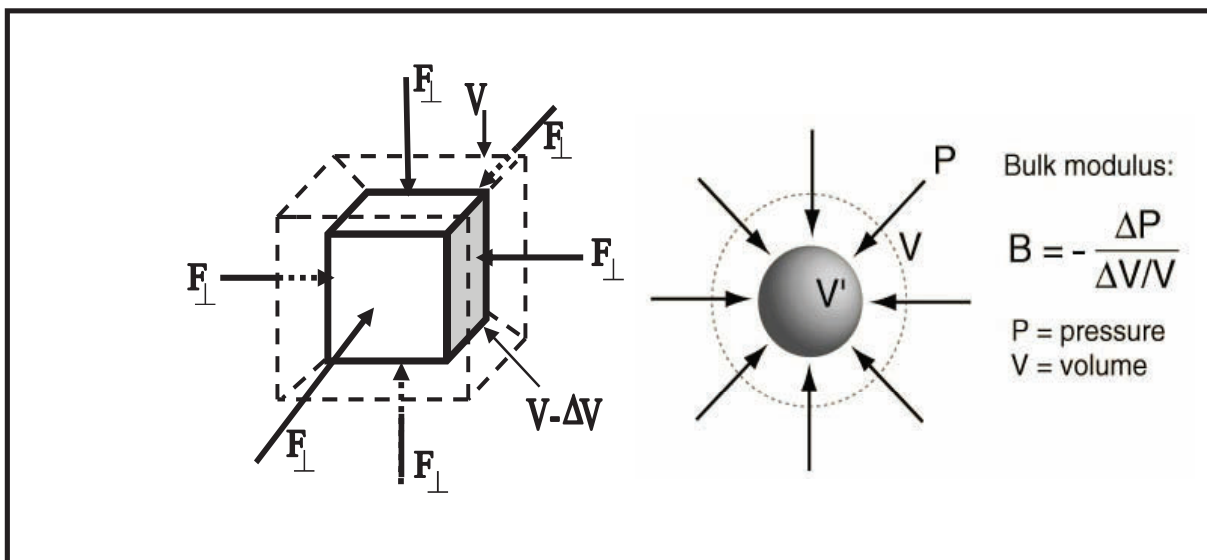


Figure (2.6)

It is important to note that solids and liquids have a bulk modulus. Liquids do not exhibit Young's modulus or shear modulus because a liquid does not sustain a shearing stress or a tensile stress. If a shearing force or a tensile force is applied to a liquid , the liquid simply flows in response .

Compressibility. K

The reciprocal of the bulk modulus is called the compressibility.

$$K = 1/B = -(1/V) \cdot \Delta V / \Delta P \quad (2.10)$$

Note : Both solids and liquids have Bulk Moduli.

Example (2.8)

A cube with 2.0 cm sides is made of material with a bulk modulus of $4.7 \times 10^5 \text{ N/m}^2$. When it is subjected to a pressure of $2.0 \times 10^5 \text{ Pa}$ the length in cm of its sides is:

Solution

$$B = \frac{\Delta p}{\frac{\Delta V}{V}} \Rightarrow \Delta V = \frac{\Delta P}{B} \cdot V$$

$$\Delta V = \frac{2 \times 10^5}{4.7 \times 10^5} \cdot (2 \times 10^{-2})^3 = 3.4 \times 10^{-5} \text{ m}^3 = 3.4 \text{ cm}^3$$

$$V - \Delta V = 8 - 3.4 = 4.6 \text{ cm}^3$$

$$L = \sqrt[3]{4.6} = 1.66 \text{ cm}$$

Example (2.9)

A wire of length 120cm and diameter 0.82mm, supported from one end, A 5.3kg in the other end. Find:

- The stress.
- The strain.
- The strain energy If $Y = 1.2 \times 10^{12} \text{ N/m}^2$ and $g = 9.80 \text{ m/sec}^2$

Solution

$$r = \frac{0.82}{2} = 4.1 \times 10^{-2} \text{ m and } m = 5.3 \text{ Kg}$$

$$\text{The stress} = \frac{F}{A} = \frac{m g}{A} = \frac{(5.3) 9.8}{\pi (4.1 \times 10^{-2})^2} = 9.835 \times 10^3 \text{ N / m}^2$$

$$\text{The strain} = \frac{\text{Stress}}{Y} = \frac{9.835 \times 10^3}{1.2 \times 10^{12}} = 8.2 \times 10^{-4}$$

Example (2.10)

A uniform wire of length 20cm density 0.78 gm / cm^3 and mass 16gm stretched by a distance 1.2mm when 8kg is supported on it , Find :

- The stress.
- Young's modulus
- The strain energy

Solution

$$\text{Volume } V = \frac{m}{\rho} = \frac{16}{7.8} = 2.05 \text{ cm}^3$$

$$\text{But } V = A \cdot L \Rightarrow A = \frac{V}{L} = \frac{2.05}{20} = 0.1 \text{ cm}^2 = 1 \times 10^{-5} \text{ m}^2$$

$$\text{Stress} = \frac{F}{A} = \frac{m g}{A} = \frac{(8) 9.80}{1 \times 10^{-5}} = 7.84 \times 10^6 \text{ N / m}^2$$

$$\text{But Stress} = Y \frac{\Delta L}{L} \Rightarrow Y = \frac{\text{Stress} \times 0.20}{1.2 \times 10^{-3}} = 1.31 \times 10^9 \text{ N / m}^2$$

2.3. Problems

Choose The Correct Answer in Each of The Followings:

1) It characterizes the response of an object to changes in a force of uniform magnitude applied perpendicularly over the entire surface of the object. An object subject to this type of deformation undergoes a change in volume but no change in shape

- A) Young's modulus
- B) Bulk modulus
- C) Stress
- D) Shear modulus

2) The material ultimately breaks as the stress is increased more than the:

- A) Elastic limit
- B) Elastic behavior
- C) Breaking point
- D) Plastic behavior

3) It returns to its original shape when the deforming forces are removed.

- A) Elastic object
- B) Inelastic object
- C) Break object
- D) Plastic object

4) The measure of the resistance to motion of the planes within a solid, parallel to each other.

- A) Young's modulus
- B) Bulk modulus
- C) Stress
- D) Shear modulus

5) Type of deformation occurs when an object is subjected to a force parallel to one of its faces (tangential force) while the opposite face is held fixed by another force

- A) Young's modulus
- B) Bulk modulus
- C) Stress
- D) Shear modulus

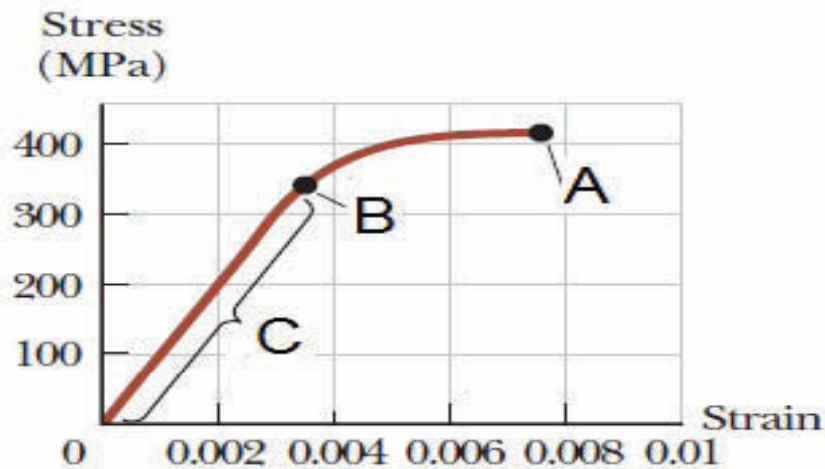
6) The measure of the degree of deformation

- A) Elastic modulus
- B) Strain
- C) Stress
- D) Force

- 7) The measure of the resistance of a solid to the change in its length.
- A) Young's modulus
 - B) Bulk modulus
 - C) Stress
 - D) Shear modulus
- 8) A 100 kg load is hung on a wire of length 4.00 m, cross sectional area $1 \times 10^{-4} \text{ m}^2$, and Young's modulus $8.0 \times 10^{10} \text{ N/m}^2$. What is its increase in length?
- A) $9.8 \times 10^{-4} \text{ m}$
 - B) $4.9 \times 10^{-4} \text{ m}$
 - C) $4.9 \times 10^{-6} \text{ m}$
 - D) $9.8 \times 10^{-6} \text{ m}$
- 9) Young's modulus can be correctly given in:
- A) N.m^2
 - B) N/m^2
 - C) N.m/s
 - D) joules
- 10) Young's modulus is a proportionality constant that relates the force per unit area applied perpendicularly at the surface of an object to:
- A) the shear
 - B) the fractional change in volume
 - C) the fractional change in length
 - D) the pressure
- 11) If the material fails to restore its original dimensions after removing the applied stress, it said to be in
- A) elastic limit
 - B) elastic behavior
 - C) breaking point
 - D) plastic behavior
- 12) Strain can be measured in:
- A) N/s^2
 - B) j.m^2
 - C) w/m
 - D) none of these

- 13) The ratio of the stress to the resulting strain is
A) Elastic modulus
B) Strain
C) Stress
D) Force
- 14) A force of 5000 N is applied outwardly to each end of a 5.0 m long rod with a radius of 34.0 mm and a Young's modulus of $125 \times 10^8 \text{ N/m}^2$. The elongation of the rod is (in mm):
A) 0.022
B) 0.0040
C) 0.11
D) 0.55
- 15) A shearing force of 50 N is applied to an aluminum rod with a length of 10 m, a cross-sectional area of $1.0 \times 10^{-5} \text{ m}^2$, and shear modulus of $2.5 \times 10^{10} \text{ N/m}^2$. As a result the rod is sheared through a distance of (in mm):
A) Zero
B) 2
C) 20
D) 200
- 16) A child slides across a floor in a pair of rubber-soled shoes. The friction force acting on each foot is (20.0 N) . The foot- print area of each shoe sole is (14.0 cm^2) , and the thickness of each sole is (5 mm) . Find the horizontal distance by which the upper and lower surfaces of each sole are offset (in mm). The shear modulus of the rubber is (3.00 MN/m^2) .
A) 0.85
B) 1.15
C) 1.66×10^2
D) 2.4×10^{-2}
- 17) Measures the resistance of solids or liquids to changes in their volume
A. Young's modulus
B. Bulk modulus
C. Stress
D. Shear modulus
E. None of them
- 18) The bulk modulus is a proportionality constant that relates the pressure acting on an object to:
A) The shear
B) The fractional change in volume
C) The fractional change in length
D) Young's modulus

Stress-strain curve for an elastic solid.



From the shown figure answer, 19, 20, 21, 22

19) Evaluate Young's modulus for the material whose stress-strain curve is shown above

- A) 1×10^5 Pa
- B) 2×10^6 Pa
- C) 2×10^{11} Pa
- D) 1×10^{11} Pa

20) The point B is

- A) Elastic limit
- B) Elastic behavior
- C) Breaking point
- D) Plastic behavior

21) The point A is

- A) Elastic limit
- B) Elastic behavior
- C) Breaking point
- D) Plastic behavior

22) The region C is

- A) Elastic limit
- B) Elastic behavior
- C) Breaking point
- D) Plastic behavior

23) The measure of the resistance of a solid to the change in its length.

- A. Young's modulus
- B. Bulk modulus
- C. Stress
- D. Shear modulus

24) Measures the resistance to motion of the planes within a solid parallel to each other.

- A. Young's modulus
- B. Bulk modulus
- C. Stress
- D. Shear modulus

Q2) The Young's modulus for bone is $1.50 \times 10^{10} \text{ Pa}$. The bone breaks if stress greater than $1.50 \times 10^8 \text{ Pa}$ is imposed on it.

(1) What is the maximum force that can be exerted on the femur bone in the leg if it has a minimum effective diameter of 2.50 cm ?

(2) If this much force is applied compressively, by how much does the 25.0 cm long bone shorten?

Solution:

(1)

$$\text{diameter} = 2r = 2.5 \text{ cm} \Rightarrow r = 1.25 \text{ cm} = 1.25 \times 10^{-2} \text{ m}$$

$$A = \pi r^2 = 3.14(1.25 \times 10^{-2})^2 = 4.91 \times 10^{-4} \text{ m}^2$$

$$\text{Stress} = \frac{F}{A} \Rightarrow 1.5 \times 10^8 = \frac{F}{4.91 \times 10^{-4}} \Rightarrow F = 1.5 \times 10^8 \times 4.91 \times 10^{-4} = 7.37 \times 10^4 \text{ N}$$

(2)

$$Y = \frac{\text{tensile stress}}{\text{tensile strain}} = \frac{\frac{F}{A}}{\frac{\Delta L}{L}} \Rightarrow Y = \frac{F \times L}{\Delta L \times A} \Rightarrow 1.5 \times 10^{10} = \frac{7.37 \times 10^4 \text{ N} \times 25 \times 10^{-2}}{\Delta L \times 4.91 \times 10^{-4}} \Rightarrow$$

$$\Delta L = \frac{7.37 \times 10^4 \text{ N} \times 25 \times 10^{-2} \text{ m}}{1.5 \times 10^{10} \text{ N/m}^2 \times 4.91 \times 10^{-4} \text{ m}^2} = 2.50 \times 10^{-3} \text{ m} = 2.50 \text{ mm}$$

Q3) We analyzed a cable used to support an actor as he swung onto the stage. Now suppose the tension in the cable is 940 N as the actor reaches the lowest point. What diameter should a 10 m long steel cable have if we do not want it to stretch more than 0.50 cm under these conditions?

if Young's modulus = $20 \times 10^{10} \text{ N/m}^2$

$$Y = \frac{\frac{F}{A}}{\frac{\Delta L}{L}} \Rightarrow Y = \frac{F \cdot L}{A \cdot \Delta L} \Rightarrow A = \frac{F \cdot L}{Y \cdot \Delta L} = \frac{(940) \times (10)}{(20 \times 10^{10}) \times (0.5 \times 10^{-2})} = 9.4 \times 10^{-6} \text{ m}^2$$

We have

$$d = 2r \text{ and } A = \pi r^2$$

$$A = \pi r^2 = 9.4 \times 10^{-6} \Rightarrow r = \sqrt{\frac{9.4 \times 10^{-6}}{\pi}} = 1.73 \times 10^{-3} \Rightarrow d = 2r \approx 3.5 \times 10^{-3} \text{ m} = 3.5 \text{ mm}$$

Q4) A steel wire of diameter 1mm can support a tension of 200N . A steel cable to support a tension of 20000N should have a diameter of what order of magnitude?

$$d = 1\text{mm} = 2r \Rightarrow r = 0.5\text{mm} = 0.5 \times 10^{-3}\text{m}$$

We have $\varepsilon = \frac{F}{A}$, $A = \pi r^2$,

$$\text{Stress} = \frac{F_1}{A_1} = \frac{F_2}{A_2} \Rightarrow \frac{F_1}{\pi r_1^2} = \frac{F_2}{\pi r_2^2} \Rightarrow \frac{0.2 \times 10^3 \text{N}}{\pi (0.5 \times 10^{-3})^2 \text{m}^2} = \frac{20 \times 10^3 \text{N}}{\pi (r_2^2) \text{m}^2} \Rightarrow$$

$$r_2^2 = \frac{(0.5 \times 10^{-3})^2 \times 20 \times 10^3}{0.2 \times 10^3} = 25 \times 10^{-6} \text{m}^2 \Rightarrow$$

$$r_2 = 5 \times 10^{-3} \text{m} \Rightarrow d = 2r_2 = 2 \times 5 \times 10^{-3} = 10 \times 10^{-3} \text{m} = 1\text{cm}$$

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جامعة الأمير سطام بن عبد العزيز
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Chapter (3)

Fluid Mechanics



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Chapter (3) Fluid Mechanics

Content

Density – Relative density – Relative density measurements - Pressure – Pascal's principle - Archimedes' Principle - Law of floatation - Continuity equation - Bernoulli's Equation – application - Venturi tube- surface tension - Surface energy – Adhesive force – Cohesive force - Surface tension and spherical shape - Contact angle - Capillarity.

Learning Objectives

1. Distinguish fluids from solids.
2. When mass is uniformly distributed, relate density to mass and volume.
3. Apply the relationship between hydrostatic pressure, force, and the surface area over which that force acts.
4. Describe how an open-tube manometer can measure the gauge pressure of a gas.
5. Identify Pascal's principle.
6. Describe Archimedes' principle.
7. Describe steady flow, incompressible flow, nonviscous flow, and irrotational flow.
8. Identify that Bernoulli's equation.
9. State the surface tension - Adhesive force – Cohesive force.

3.1 Fluids

Matter in the solid state generally offers considerable resistance to all changes in shape. Liquids and gases, in contrast, do not have rigid structure or form. These states of matter – which together we call fluids, are easily altered in shape.

Liquids generally have very low compressibility so they deform in shape without appreciable change in volume. Gases on the other hand, readily change volume and expand to fill completely any container. Neither liquids nor gases can permanently sustain a shearing stress.

It is customary to classify matter into solids and fluids.

A fluid is a substance that can flow. Hence the term fluid includes liquids and gases. Generally, a fluid is a collection of molecules that are randomly arranged and held together by weak cohesive forces and by forces exerted by walls of a container. Fluid static is the study of fluids at rest.

Density (ρ): $\rho = M / V$ its unit kg/m^3 & Dimension $[\text{ML}^{-3}]$

Relative Density (ρ_r) the ratio between the density of material to the density of the water (same volume).

$$\rho_{re} = \rho_{matter} / \rho_{water}$$

3.2 Pressure

When fluids are at rest, there are no shear forces. The only stress that can be exerted on an object submerged in a static fluid is one that tends to compress the object from all sides. In other words, the force exerted by a static fluid on an object is always perpendicular to the surfaces of the object. See Figure (3.1)

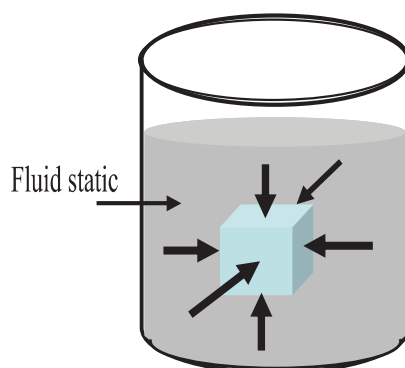


Figure (3.1)

It is convenient, therefore, to describe the force F acting on a fluid by specifying the pressure p , which is defined as the magnitude of the normal force per unit surface area. If F is the magnitude of the normal force exerted on a surface area A at a given level, then the pressure p of the fluid at this level is defined as the ratio F/A ; that is :

$$P = F / A \quad \text{Pascal} \quad (3.1)$$

The Pressure P is a scalar quantity.

$$1 \text{ Pascal} = 1 \text{ N/m}^2$$

$$1 \text{ atm.} = 1.013 \times 10^5 \text{ Pa} = 760 \text{ torr}$$

$$1 \text{ bar} = 10^5 \text{ Pa}$$

3.3: Variation of Pressure with Depth

Assume we have an imaginary cylinder of incompressible liquid at rest of density ρ and cross-sectional area A and depth h from surface see Figure (3.2).

The pressure of liquid on the bottom face is P and on top face is P_0 .

The upward force on bottom face is $F_1 = P A$

The downward force on the top face is $F_2 = -P_0 A$

The weight of the cylinder is $Mg = -\rho A g(y_2 - y_1) = -\rho A h g$

The cylinder is in equilibrium, net force = zero

$$P A - P_0 A + Mg = 0 \quad \text{OR} \quad P A - P_0 A + A\rho gh = 0 \rightarrow P = P_0 + \rho gh \quad (3.2)$$

P_0 is atmospheric pressure, $P_0 = 1 \text{ atm} = 1.013 \times 10^5 \text{ Pa}$ or N/m^2

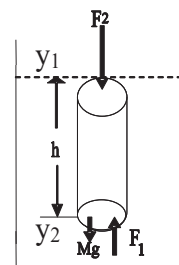


Figure (3.2)

Remark:

The Pressure with depth $P = P_0 + \rho gh$

The Pressure at the depth h below sea level $P = P_0 + \rho gh$

The Pressure at the height h high above sea level $P = P_0 - \rho gh$



Checkpoint 1

The figure shows four containers of olive oil. Rank them according to the pressure at depth h , greatest first.

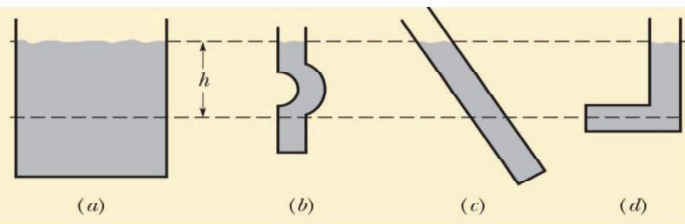


Figure (3.3)

Answer: All the pressures will be the same. All that matters is the distance h , from the surface to the location of interest, and h is the same in all cases.

Example (3.1)

The mattress of a water bed is 2.00m long by 2.00m wide and 30.0cm deep. Find the weight of the water in the mattress. Assuming Density of water $\rho = 1000 \text{ kg / m}^3$

Answer

Density of water is $\rho = M / V \Rightarrow M = \rho V = (1000)(2)(2)(0.3) = 1.2 \times 10^3 \text{ kg}$

weight of the water $= Mg = (1.2 \times 10^3)(9.8) = 1.18 \times 10^4 \text{ (kg.m / sec}^2\text{) or N}$

Example (3.2)

What is the pressure due to water at a depth of 7.5 Km below sea level?

The water density $\rho_w = 1.025 \times 10^3 \text{ Kg / m}^3$.

Answer

$$P = P_0 + \rho gh$$

$$P = 1.013 \times 10^5 + 1.025 \times 10^3 \times 9.8 \times 7.5 \times 10^3$$

$$P = 7.53 \times 10^7 \text{ N / m}^2 = 75.3 \text{ MPa}$$

Example (3.3)

What is the pressure at a point 2000m high above sea level assuming that the density of air is approximately constant and $\rho_{\text{air}} = 1.22 \text{ Kg / m}^3$?

Answer

$$P = P_0 + \rho_{\text{air}} gh$$

$$P = 1.013 \times 10^5 - 1.22 \times 9.8 \times 2000$$

$$P = 7.74 \times 10^4 \text{ N / m}^2$$

3.4 Measurement of Pressure

3.4.1 The Mercury Barometer

The mercury barometer is a long glass tube that has been filled with mercury, as in Figure (3.4). The space above the mercury column contains only mercury vapor whose pressure is so small at ordinary temperatures that it can be neglected. It is easily shown (see Equation (3.2) that the atmospheric pressure P_0 is :

$$p_0 = \rho gh$$

Most pressure gauges use atmospheric pressure as a reference level and measure the difference between the actual

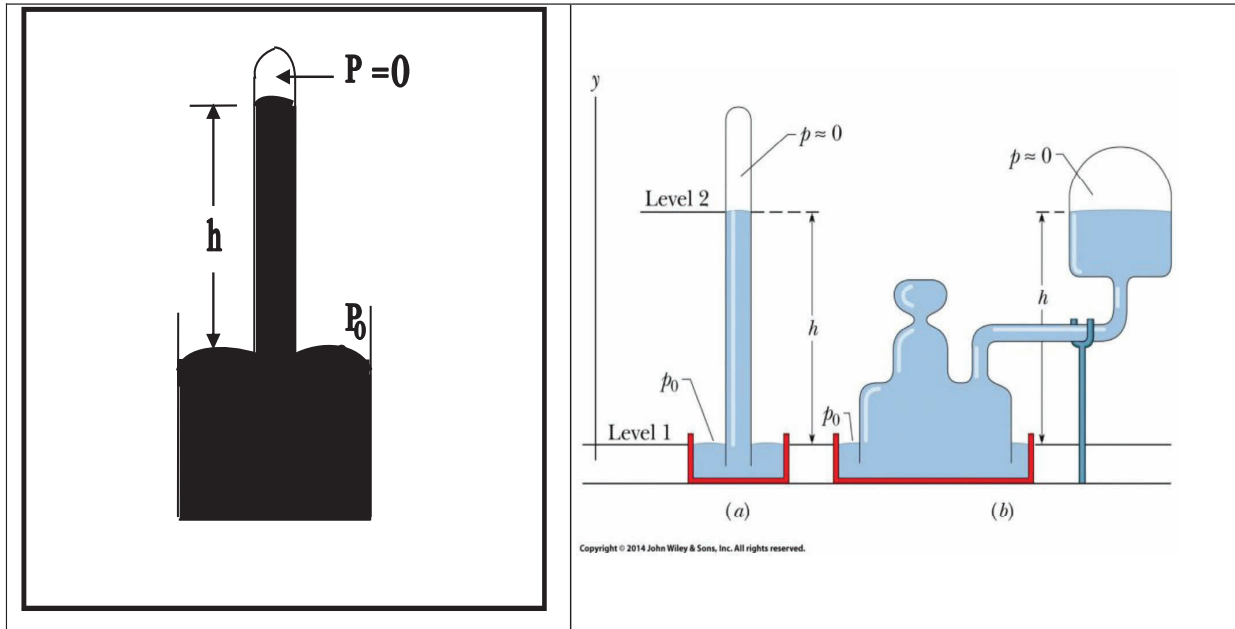


Figure (3.4)

pressure and atmospheric pressure, called the gauge pressure. The actual pressure at a point in a fluid is called the absolute pressure. Gauge pressure is given either above or below atmospheric pressure. A gauge that reads pressures below atmospheric pressure is usually called a vacuum gauge.

The pressure of the atmosphere at any point is numerically equal to the weight of a column of air of unit cross-sectional area extending from that point to the top of the atmosphere. The atmospheric pressure at a point, therefore, decreases with altitude. There are variations in atmospheric pressure from day to day since the atmosphere is not static. The mercury column in the barometer will have a height of about 76 cm at sea level, varying with atmospheric pressure. A pressure equivalent to that exerted by exactly 76 cm of mercury at 0°C under standard gravity, $g = 9.80665 \text{ m/sec}^2$, is called one atmosphere (1 atm). The density of mercury at this temperature is 13595 kg/m^3 . Hence, one atmosphere is equivalent to

$$p_0 = \rho gh$$

$$1 \text{ atm} = 13595 \times 9.80665 \times 0.76$$

$$1 \text{ atm} = 1.013 \times 10^5 \text{ N/m}^2$$

note that ; $(\text{N/m}^2 \equiv \text{Pa})$

It must be noted that for a given pressure, the height h of the mercury column does not depend on the cross-sectional area of the vertical tube.

For a given pressure P_0 , the height depends on the value of the acceleration due to gravity g at the location of the barometer and on the density ρ of the mercury.

3.4.2 The Open – Tube Manometer

The open tube manometer, Figure (3.5) , measure gauge pressure . It consists of U-shaped tube containing a liquid, one end of the tube being open to the atmosphere and the other end being connected to the system (tank) whose pressure P we want to measure (unknown). The pressures at points A and B must be the same. Equating the unknown pressure P to the pressure at point B , we see that

$$P = P_0 + \rho gh$$

Or
$$P - P_0 = \rho gh$$

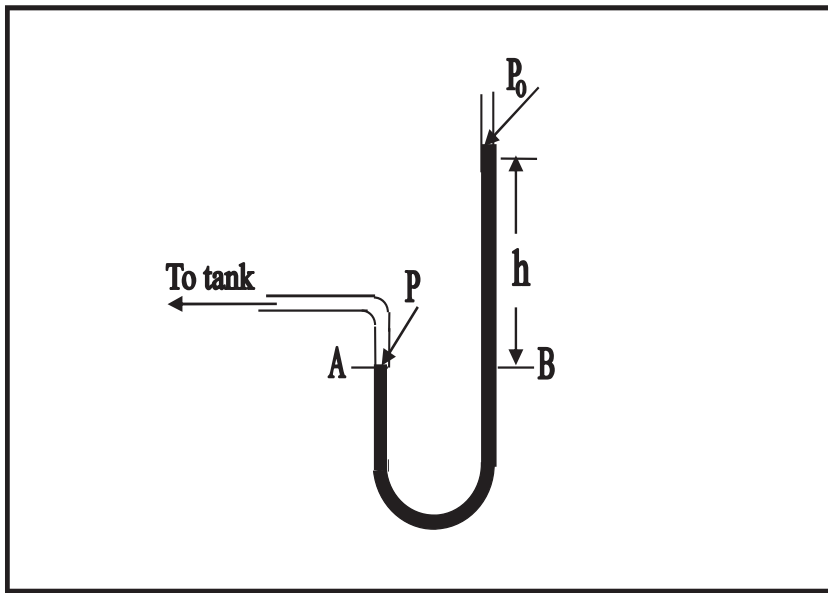


Figure (3.5)

The difference $(P - P_0)$ is called the **gauge pressure** , while the pressure P is called the **absolute pressure**. For example the pressure you measured in your bicycle tire is gauge pressure .

3.5 Pascal's Principle:

Figure (3.6) shows a liquid in a cylinder that is fitted with a piston to which we may apply an external pressure P_0 . The pressure P at any arbitrary point C a distance h below the upper surface of the liquid is given by Equation (3.2) , or

$$P = P_0 + \rho gh$$

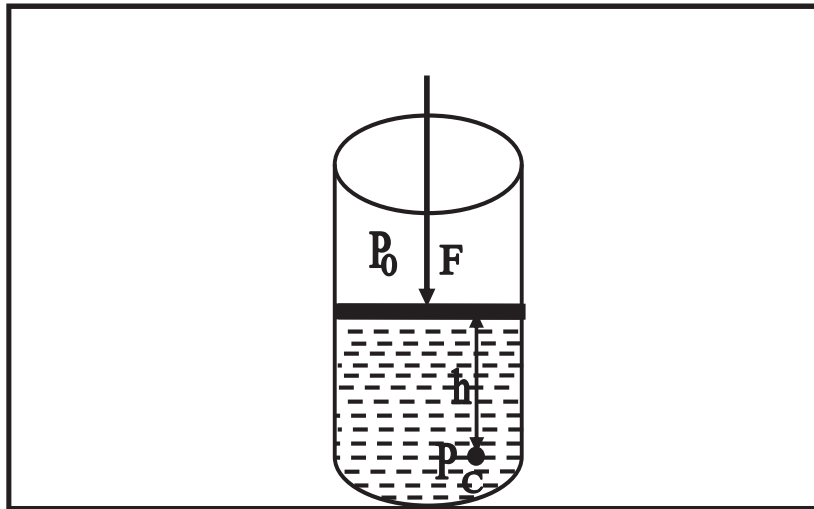


Figure (3.6)

Let's increase the external pressure by an arbitrary amount ΔP_0 . Since liquids are virtually incompressible, the density ρ in the above equation remains essentially constant during the process. The equation shows that, the change in pressure ΔP at the arbitrary point C is equal to ΔP_0 . This result was stated by the French scientist Pascal and is called Pascal's principle which usually given as follows : Pressure applied to an enclosed fluid is transmitted undiminished to every portion of the fluid and the walls of the containing vessel. An important application of Pascal's law is the hydraulic press illustrated in Figure (3.7)

A force of magnitude F_i is applied to a small piston of surface area A_i . The pressure is transmitted through an incompressible liquid to a larger piston of surface area A_o . Because the pressure must be the same on both sides, we have : $P = F_i / A_i = F_o / A_o$ or

$$F_o = \frac{A_o}{A_i} F_i \quad \& \quad d_o = d_i \frac{A_i}{A_o} \quad (3.3)$$

Therefore, the force F_o is greater than the force F_i by the factor A_o / A_i . Hydraulic brakes, car lifts, hydraulic jacks, and forklifts all make use of this principle.

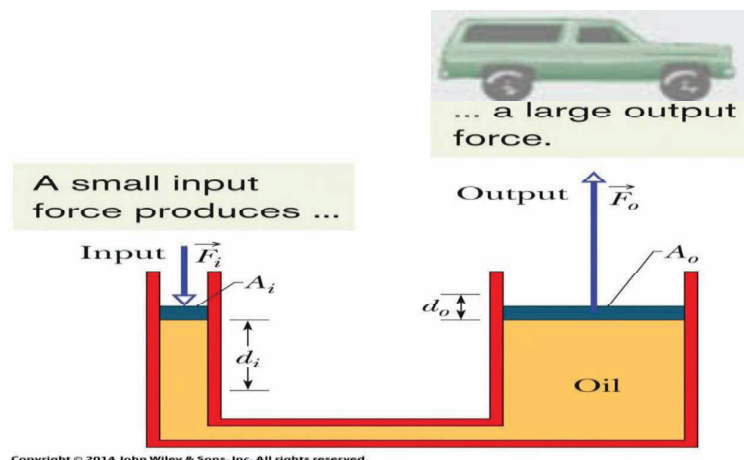


Figure (3.7)

3.6 Archimedes' Principle

The magnitude of the buoyant force always equals the weight of the fluid displaced by the object.

If a body is totally or partially immersed in a fluid, the buoyant force will equal to the weight of displaced fluid

❖ Verification of Archimedes principle

If a cube of height h immersed in a liquid of density ρ_{fluid} , Figure (3.8), the pressure at top and bottom faces are P_1 and P_2 respectively, Where $P_2 = P_1 + \rho g h$

1. The pressure at the bottom of the cube causes an upward force equals $P_2 A$.
2. The pressure at the top of the cube causes a downward force equals $P_1 A$.

Where A is the area of face of the cube,

3. the resultant force is the buoyant force “B”

$$B = (P_2 - P_1) A = \rho_{\text{fluid}} g h A = \rho_{\text{fluid}} g V$$

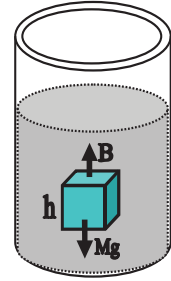


Figure (3.8)

V is the volume of the liquid displaced by the cube.

Because the product $(\rho V) = \text{mass of liquid}$, so

$$B = Mg = \rho_{\text{fluid}} g V \quad (3.4)$$

Or

$$F_B = F_2 - F_1 \quad (3.5)$$

But, as known,

$$F_2 = P_2 A \quad \text{and} \quad F_1 = P_1 A$$

Also,

$$P_2 = P_0 + \rho g h_2 \quad \text{and} \quad P_1 = P_0 + \rho g h_1$$

Therefore,

$$F_2 = (P_0 + \rho g h_2) A \quad \text{and} \quad F_1 = (P_0 + \rho g h_1) A$$

Then

$$\begin{aligned} F_B &= (P_0 + \rho g h_2) A - (P_0 + \rho g h_1) A \\ &= P_0 A + \rho g h_2 A - P_0 A - \rho g h_1 A = \rho g h_2 A - \rho g h_1 A \\ F_B &= \rho g A (h_2 - h_1) = \rho g A H, \text{ where } H = h_2 - h_1 \end{aligned}$$

$$F_B = \rho g A H = \rho g V = mg, \text{ where } m = \rho V$$

Special cases:**Case 1: Totally Submerged Object**

When an object is totally submerged in a fluid of density ρ_{fluid} , the magnitude of the upward buoyant force is:

$$\mathbf{B} = \mathbf{Mg} = \rho_{fluid} g V_{object}$$

Where: V_{object} is the volume of the object. If the object has a mass $\mathbf{M}$ and density ρ_{object} , its weight (gravitational force) is equal to:

$$F_g = \mathbf{Mg} = \rho_{object} g V_{object}$$

and the net force on it is:

$$F_{net} = F_g - B = (\rho_{object} - \rho_{fluid}) g V_{object}$$

1. Hence, if $\rho_{object} < \rho_{fluid}$, then the downward gravitational force is less than the buoyant force, and the object float, Figure (3.9).

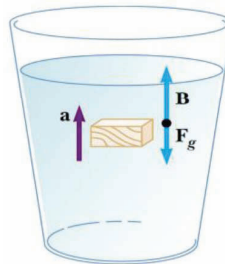


Figure (3.9)

2. If the $\rho_{object} > \rho_{fluid}$, then the upward buoyant force is less than the downward gravitational force, and the object sinks, Figure (3.10).

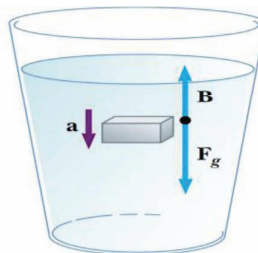


Figure (3.10).

3. If the $\rho_{object} = \rho_{fluid}$, the net force on the object is zero and it remains in equilibrium. Thus, the direction of motion of an object submerged in a fluid is determined only by the densities of the object and the fluid.

Case 2: Floating Object

Now consider an object of volume V_{object} and density $\rho_{\text{object}} < \rho_{\text{fluid}}$ floating on the surface of a fluid—that is, an object that is only partially submerged, Figure (3.11).

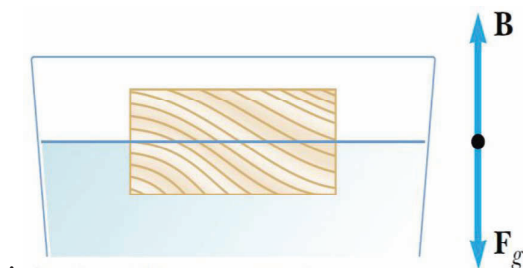


Figure (3.11)

In this case, the upward buoyant force is balanced by the downward gravitational force acting on the object. If V_{fluid} is the volume of the fluid displaced by the object (this volume is the same as the volume of that part of the object that is beneath the surface of the fluid), the buoyant force has a magnitude

$$B = \rho_{\text{fluid}} g V_{\text{fluid}}$$

Because the weight of the object is

$$F_g = \rho_{\text{object}} g V_{\text{object}}$$

and because $F_g = B$, we see that

$$\rho_{\text{object}} g V_{\text{object}} = \rho_{\text{fluid}} g V_{\text{fluid}}$$

or,

$$\frac{\rho_{\text{fluid}}}{\rho_{\text{object}}} = \frac{V_{\text{object}}}{V_{\text{fluid}}}$$

Example (3.5)

In a car lift used in a service station, compressed air exerts a force on a small piston that has a circular cross section and a radius of 5.00cm. This pressure is transmitted by a liquid to a piston that has a radius of 15.0cm. What force must the compressed air exert to lift a car weighing 13,300N? What air pressure produces this force?

Solution

Using the Pascal's principle,

$$F_1 = \frac{A_1}{A_2} F_2 = \frac{\pi(0.05)^2}{\pi(0.15)^2} (1.33 \times 10^4) = 1.48 \times 10^3 \text{ N}$$

The necessary pressure of the compressed air is

$$P_1 = \frac{F_1}{A_1} = \frac{1.48 \times 10^3}{\pi(0.05)^2} = 1.88 \times 10^5 \text{ N / m}^2$$

Example (3.6)

What fraction of the total volume of an iceberg is exposed?

The density of ice is $\rho_{\text{ice}} = 0.92 \text{ g/cm}^3$ and that of sea water is $\rho_{\text{water}} = 1.03 \text{ g/cm}^3$.

Solution

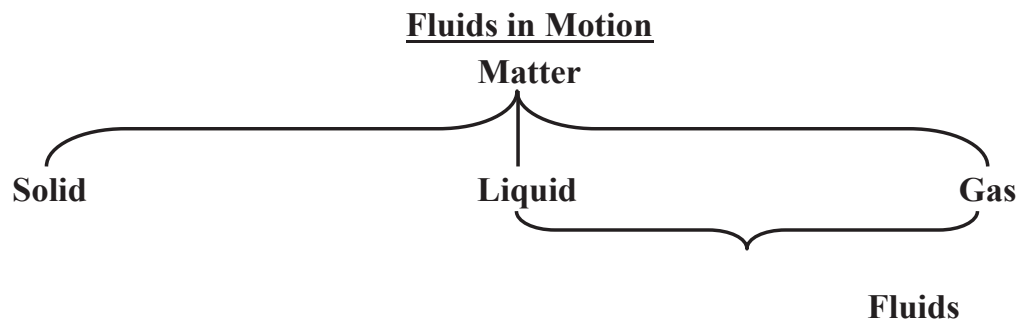
Weight of ice is $W_{\text{ice}} = \rho_{\text{ice}} V_{\text{ice}} g$ the buoyant force of water is $B = \rho_{\text{water}} V_{\text{water}} g$

$$\rho_{\text{water}} V_{\text{water}} g = \rho_{\text{ice}} V_{\text{ice}} g \Rightarrow \rho_{\text{water}} V_{\text{water}} = \rho_{\text{ice}} V_{\text{ice}}$$

$$\frac{V_{\text{water}}}{V_{\text{ice}}} = \frac{\rho_{\text{ice}}}{\rho_{\text{water}}} = \frac{0.92}{1.03} = 0.89$$

The volume of ice exposed in air is $100\% - 89\% = 11\%$

3.7 Fluid Dynamics



3.7.1 General Concepts of Fluids Flow

Fluid dynamics is the study of fluids in motion. Fluid flow can be steady (laminar) or non-steady. When the fluid velocity v at any given point is constant in time, the fluid motion is said to be steady. In non-steady flow, the velocities v are function of the time.

- Fluid flow can be rotational or irrotational. If the element of fluid at each point has no net angular velocity about that point, the fluid flow is irrotational.
- Fluid flow can be compressible or incompressible. Liquids can usually be considered as flowing incompressibly. But even a highly compressible gas may sometimes undergo unimportant changes in density. Its flow is then practically incompressible.
- Finally fluid flow can be viscous or nonviscous. Velocity in fluid motion is the analog of friction in the motion of solids. Viscosity introduces tangential forces between layers of fluid in relative motion and results in dissipation of mechanical energy.

We shall confine our discussion of fluid dynamics for the Ideal fluid flow, therefore the following four concepts are considered :

1. **The fluid is steady.** In steady (laminar) flow, the velocity of the fluid at each point remains constant.
2. **The fluid is nonviscous.** In this case, internal friction is neglected. An object moving through the fluid experiences no viscous force.
3. **The fluid is incompressible.** The density of an incompressible fluid is constant.
4. **The flow is irrotational.** In irrotational flow, the fluid has no angular momentum about any point.

3.7.2 Streamlines

The path taken by a fluid particle under a steady flow is called a streamline. The velocity of the particle is always tangent to the streamline, as in Fig (3.12). Consider the point p within the fluid, since v at P does not change in time, every particle arriving at P will pass on with the same speed in the same direction. The same is true about the point Q and R . The curve in Figure (3.12) is called streamline.

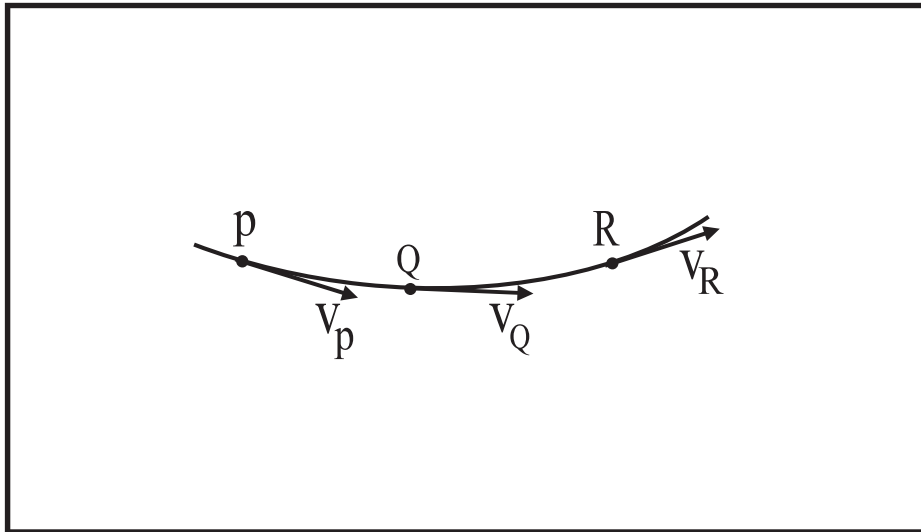


Figure (3.12)

In steady flow, streamlines cannot intersect (otherwise a particle reaching the intersection could follow either of two paths and the flow would not be steady). Therefore, in steady flow, streamlines illustrate a fixed pattern of the flow. In principle we can draw a streamline through every point in the fluid. Let us assume steady flow and select a finite number of streamlines to form a bundle, this tubular region is called a stream tube or a tube of flow. Thus a stream tube is a region in a fluid bounded by streamlines, as seen in Figure (3.12). In steady flow, a particle within stream tube cannot pass outside the tube (because its streamline would then have to intersect a streamline bounding the stream tube).

In summery

- The Solid has a fixed shape, which it tends to retain.
- But, Fluids have no fixed shape and they take the shape of the container.

3.7.3 Streamline flow

It is the motion of a fluid in which every particle in the fluid follows the same path as the previous particle.

- We shall assume that the fluids are :

- 1) Incompressible
- 2) Have no internal frictions

3.8 The Continuity Equation

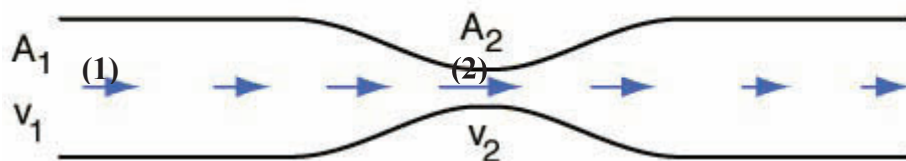


Figure (3.13)

In Figure (3.13), the velocity of the fluid inside the tube of flow may have different magnitudes at different points (although it parallel to the tube at any point) Let the speed be v_1 for fluid particles at (1) and v_2 for fluid particles at (2) . Let A_1 and A_2 be that cross- sectional areas of the tube perpendicular to the streamlines at the points (1) and (2), respectively. In the time interval Δt a fluid element travels approximately the distance $v\Delta t$.

Then the mass of fluid Δm_1 crossing A_1 in the time interval Δt is :

$$\Delta m_1 = \rho_1 A_1 V_1 \Delta t$$

The mass of fluid Δm_2 crossing A_2 in the same time interval Δt is :

$$\Delta m_2 = \rho_2 A_2 V_2 \Delta t$$

where ρ_1 and ρ_2 are the fluid densities at (1) and (2) , respectively . Because the fluid is incompressible $\rho_1 = \rho_2 = \rho$ and because the flow is steady ,then, $\Delta m_1 = \Delta m_2$

Then

$$A_1 V_1 = A_2 V_2 = \text{constant} \quad (3.6)$$

Or

$$AV = \text{constant}$$

This expression is **called the equation of continuity for fluids** .

It states that :

The product of the area and the speed at all points along a pipe is constant for an incompressible fluid .

Equation (3.6) shows that the speed is high where the tube is constricted (small A) and low where the tube is wide (large A) . The product Av gives the volume flux or flow rate and it has the dimensions of volume per unit time

The condition $AV = \text{constant}$ is equivalent to the statement that the volume of fluid that enters one end of a tube in a given time interval equals the volume leaving the other end of the tube in the same time interval if no leaks are present .

OR

It is the volume of fluid passes a certain cross section per unit time.

$$\text{Volume Flow Rate} = \frac{\text{Volume}}{\text{Time}} = \frac{\text{Area} * \text{Distance}}{\text{Time}} = \frac{\text{Area} * \text{Velocity} * \text{Time}}{\text{Time}}$$

$$Q = \frac{A \times v \times t}{t} \quad \Rightarrow \quad R = A v$$

$$Q = A_1 v_1 = A_2 v_2 \quad (\text{Continuity Equation})$$

The amount of fluid that enter from the first slot (1) is equal to the amount of fluid that comes out of the other slot (2) during the same time.

* As the section is narrow the liquid flows faster.



Checkpoint 3

The figure shows a pipe and gives the volume flow rate (in cm^3/s) and the direction of flow for all but one section. What are the volume flow rate and the direction of flow for that section?

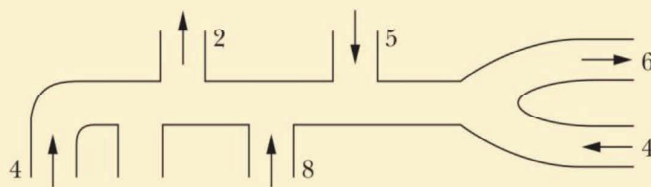


Figure (3.14)

Answer: 13, out

3.9 Bernoulli's Equation

“The relation between P , ρ , v and h (height) and their ability to describe fluids in motion”

When a fluid moves through a region where its speed and /or elevation above the Earth's surface changes, the pressure in the fluid varies with these changes. Bernoulli's equation is a general expression that relates the pressure difference between two points in a flow tube to both velocity changes and elevation changes.

Consider the flow of a segment of a nonviscous, steady, incompressible flow of a fluid through a nonuniform pipeline or tube of flow shown in Figure (3.15).

At the beginning of the time interval Δt , the segment of fluid consists of the gray portion (portion 1) at the left and the uncolored portion at the upper right. During the time interval Δt , the left end of the segment moves to the right by a distance s_1 . At the same time, the right end of the segment moves to the right through a distance s_2 , so that the volume element,

$$A_1 s_1 = A_2 s_2$$

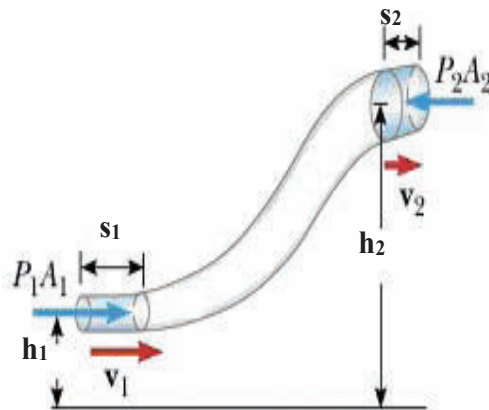


Figure (3.15)

The work done on the system by the resultant force is determined as follows :

1- The work done at point 1 to push the entering fluid (input) into the tube is the work done on the system by the pressure force F_1 is given by

$$W_1 = P_1 A_1 s_1 \quad (3.7)$$

2- The work done at point 2 to push forward the fluid out the tube (output) is the work done by the system by the pressure force F_2 is given by

$$W_2 = - P_2 A_2 s_2 \quad (3.8)$$

- Net Work = $W_1 - W_2$
- Net Work = $F_1 s_1 - F_2 s_2 = P_1 A_1 s_1 - P_2 A_2 s_2$

- Since, $\text{the Volume} = A s = A_1 s_1 = A_2 s_2$
- So, $\text{Work} = (P_1 - P_2) V$
- Change in kinetic energy $\Delta E_K = \frac{1}{2} m v_2^2 - \frac{1}{2} m v_1^2 = \frac{1}{2} m (v_2^2 - v_1^2)$
- Change in potential energy $\Delta E_P = m g h_2 - m g h_1 = m g (h_2 - h_1)$
- **The work done on the system = the increase in kinetic and potential energy.**
- $(P_1 - P_2) V = \frac{1}{2} m (v_2^2 - v_1^2) + m g (h_2 - h_1)$
- Since, the volume $V = \frac{m}{\rho}$
- $(P_1 - P_2) = \frac{1}{2} \frac{m}{V} (v_2^2 - v_1^2) + \frac{m}{V} g (h_2 - h_1)$
- $P_1 + \frac{1}{2} \rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2} \rho v_2^2 + \rho g h_2$

Where:

P_1 : Pressure

$\frac{1}{2} \rho v_1^2$: Kinetic Energy per unit volume

$\rho g h_1$: Potential Energy per unit volume

Energy per volume before = Energy per volume after

- So, $P + \frac{1}{2} \rho v^2 + \rho g h = \text{Constant} \quad (3.9) \quad \Leftarrow \quad \text{Bernoulli's Equation}$
- Where, P : is the absolute pressure, ρ : is the density of the fluid
- Absolute Pressure (P) = Gauge Pressure (P_g) + Atmospheric Pressure (P_a)

3.9.1 Applications

3.9.1.1: When the liquid is stationary; (Both v_1 and v_2 are equal zero).

From Bernoulli: $P_1 + \frac{1}{2} \rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2} \rho v_2^2 + \rho g h_2$

$$P_1 + \rho g h_1 = P_2 + \rho g h_2$$

So, $(P_2 - P_1) = \rho g (h_1 - h_2)$

3.9.1.2: Torricelli's equation :

When there is no change in pressure ($P_1 = P_2 = P_0$)

- A liquid emerges from a hole (orifice), near the bottom of an

3.9.1.2.1 open tank. see the Figure (3.16)

- The speed (v_2) at the top is equal zero.
- The pressure at both top and at the orifice is equal to the atmospheric pressure.

$$P_1 = P_2 = P_a = P_0, \quad v_2 = 0$$

$$\text{- From Bernoulli:} \quad P_1 + \frac{1}{2} \rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2} \rho v_2^2 + \rho g h_2$$

$$\text{- } \frac{1}{2} \rho v_1^2 + \rho g h_1 = \rho g h_2$$

- So,

$$v_1^2 = 2g(h_1 - h_2) = 2gh \Rightarrow$$

$$v_1 = \sqrt{2gh} \quad (3.10)$$

- Where,

A : is the cross section area of the orifice

h : is the water level above the orifice

$$\text{The rate of flow from the orifice} \quad R = A v = A \sqrt{2gh}$$

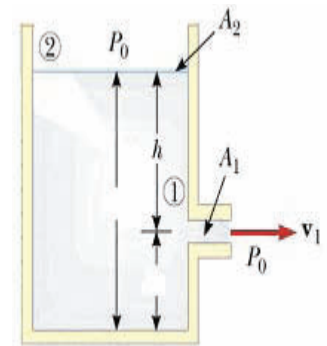


Figure (3.16)

Example (3.7)

An Open tank containing a liquid has a hole in its side. The hole is located at $h = 11.48$ m below the water level and open to the atmosphere $P_1 = P_2 = P_{atm} = 1.013 \times 10^5$ pa . We can make the following approximation: $V_2 = 0$.

Calculate the speed V_1 of the water leaking from the hole

Solution

$$v_1 = \sqrt{2gh} = \sqrt{2 \times 9.8 \times 11.48} = 15 \text{ m/s}$$

3.9.1.2.2 tank is closed, See the Figure (3.17)

$$v_2 = 0, P_1 = P_0, \quad h = h_2 - h_1$$

From Bernoulli's equation:

$$P_1 + \frac{1}{2} \rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2} \rho v_2^2 + \rho g h_2$$

$$v_1 = \sqrt{\frac{2(P_2 - P_0)}{\rho} + 2gh} \quad (3.11)$$

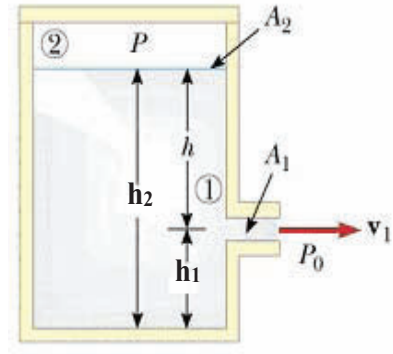


Figure (3.17)

Example (3.8)

A closed tank containing a liquid with a density $\rho = 980 \text{ kg/m}^3$ has a hole in its side . The hole is located at $h = 15.65 \text{ m}$ below the water level and open to the atmosphere $P_1 = P_{atm} = 1.013 \times 10^5 \text{ pa}$.

We can make the following approximations: $P_2 = 0$ and $V_2 = 0$.

Calculate the speed V_1 of the water leaking from the hole

Solution

$$v_1 = \sqrt{\frac{2(P_2 - P_0)}{\rho} + 2gh} = \sqrt{\frac{2(-1.013 \times 10^5)}{980} + 2 \times 9.8 \times 15.65} = 10 \text{ m/s}$$

3.9.1.3 The Venturi effect, See the Figure (3.18)

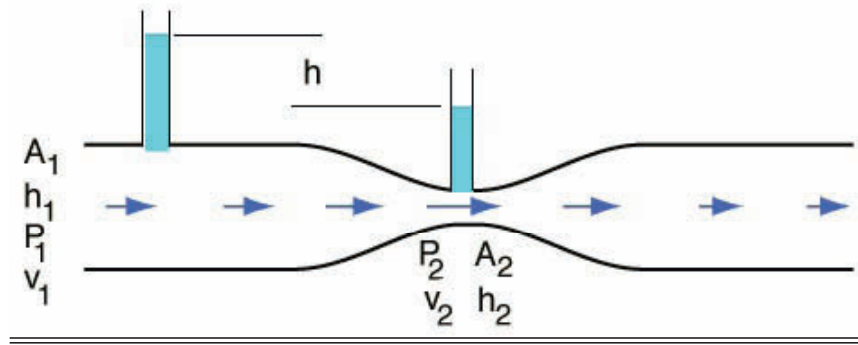


Figure (3.18)

- That describes the motion of fluid through constriction.
- $h_1 = h_2$
- From Bernoulli: $P_1 + \frac{1}{2} \rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2} \rho v_2^2 + \rho g h_2$
- $P_1 + \frac{1}{2} \rho v_1^2 = P_2 + \frac{1}{2} \rho v_2^2$
- From Continuity equation: $A_1 v_1 = A_2 v_2$, $v_1 = \frac{A_2}{A_1} v_2$

$$v_2 = A_1 \sqrt{\frac{2(P_1 - P_2)}{\rho(A_1^2 - A_2^2)}} \quad (3.12)$$



Checkpoint 4

Water flows smoothly through the pipe shown in the figure, descending in the process. Rank the four numbered sections of pipe according to (a) the volume flow rate R_V through them, (b) the flow speed v through them, and (c) the water pressure p within them, greatest first.

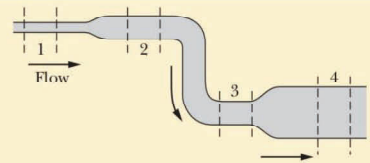


Figure (3.19)

Answer: (a) all the same volume flow rate

(b) 1, 2 & 3, 4

(c) 4, 3, 2, 1

Example (3.9)

A pipe, see the Figure (3.20), has a diameter of 16 cm at point 1 ($P_1 = 200$ KPa) and 10 cm at point 2 that is 6 m higher than portion 1. When oil of density 800 kg/m^3 flows in this pipe at a rate of $0.03 \text{ m}^3/\text{s}$. Find the pressure at point 2?

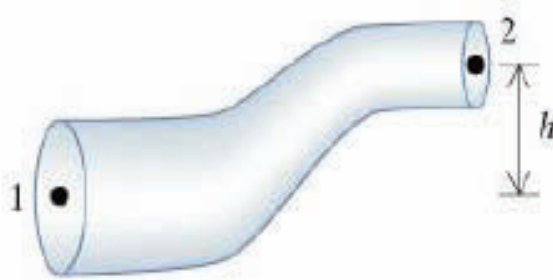


Figure (3.20)

Solution

$A_1 v_1 = A_2 v_2 = 0.03$, then

$$v_1 = 0.03 / \pi (0.08)^2 = 1.49 \text{ m/s}, \quad v_2 = 0.03 / \pi (0.05)^2 = 3.82 \text{ m/s}$$

From Bernoulli's Equation

$$P_1 + \frac{1}{2} \rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2} \rho v_2^2 + \rho g h_2$$

$$P_2 = P_1 + \frac{1}{2} \rho (v_1^2 - v_2^2) + \rho g (h_1 - h_2)$$

$$= 2 \times 10^5 + \frac{1}{2} 800 \{ (1.49)^2 - (3.82)^2 \} + 800 \times 9.8 \times 6$$

$$= 1.48 \times 10^5 \text{ Pa.}$$

Example (3.10)

A Venturi meter reads height $h_1 = 30$ cm, and $h_2 = 10$ cm. Find the velocity of flow in the pipe.

$A_1 = 7.85 \times 10^{-3} \text{ m}^2$ and $A_2 = 1.26 \times 10^{-3} \text{ m}^2$.

Solution

$$v_1 = A_2 [2gh / (A_1^2 - A_2^2)]^{1/2}$$

$$= 1.26 \times 10^{-3} [2 \times 20 \times 10^{-2} \times 9.8 / (7.85 \times 10^{-3})^2 - (1.26 \times 10^{-3})^2]^{1/2}$$

$$= 0.322 \text{ m/s}$$

Example (3.11)

A horizontal pipe has a constriction in it, as shown in the figure (3.21). At point 1 the diameter is 6 cm, while at point 2 it is only 2 cm. At point 1, $v_1 = 2 \text{ m/s}$ and $p_1 = 180 \text{ KPa}$. Calculate v_2 and p_2 if the density of water is 1000 kg/m^3 .

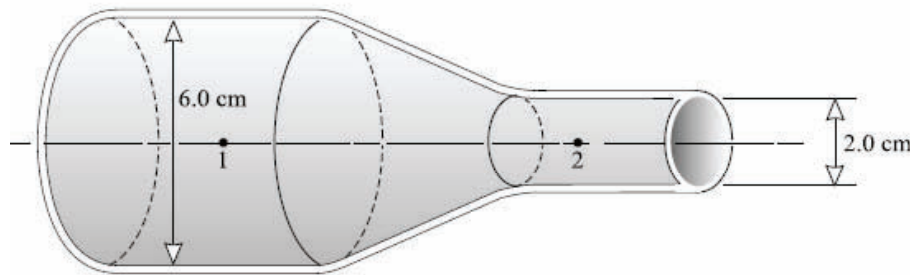


Figure (3.21)

From Bernoulli Equation:

$$P_1 + \frac{1}{2} \rho v_1^2 + \rho g y_1 = P_2 + \frac{1}{2} \rho v_2^2 + \rho g y_2 \Rightarrow y_1 = y_2$$

$$P_1 + \frac{1}{2} \rho v_1^2 = P_2 + \frac{1}{2} \rho v_2^2 \quad ; \quad R = A_1 \times v_1 = A_2 \times v_2 \text{ (Continuity Equation)}$$

$$v_2 = \frac{A_1 \times v_1}{A_2} = \frac{\pi \left(\frac{6}{2} \times 10^{-2} \right)^2}{\pi \left(\frac{2}{2} \times 10^{-2} \right)^2} \times 4 = 18 \text{ m/s}$$

$$180 \times 10^3 + \frac{1}{2} \times 100 (4)^2 = P_2 + \frac{1}{2} \times 100 (18)^2 \Rightarrow$$

$$P_2 = 20000 \text{ pa}$$

3.10 Viscosity:

In the previous sections of this chapter our discussion has been limited by assuming ideal fluids in which there is no internal friction. In real fluids there exist internal friction between adjacent moving layers of the fluid. Viscosity may be thought of as the internal friction of a fluid. Because of viscosity, a force must be exerted to cause one layer of a fluid to slide past another, or to cause one surface to slide past another if there is a layer of fluid between the surfaces. Both liquids and gases exhibit viscosity, although liquids are much more viscous than gases.

Viscosity can be defined as the resistance to flow a liquid. The flow process is one which involves molecules sliding past each other under the influence of some applied stress. The rate of flow will depend upon:

- the magnitude of the stress,
- the shape of the molecules, and
- the magnitude of the forces of intermolecular attraction

Thus, molecules approximately spherical in shape will more readily slide each other than, for example, long chain polymer molecules. Also molecules among which attractive forces are weak will flow more easily than molecules strongly bound to each other.

When a liquid moves with a steady speed over a solid the motion is quite slow then it is observed that the layer of the liquid in contact with the solid is more or less stationary. In other words the velocity of the liquid along the surface of the solid is zero and the velocity of any other layer of the liquid will be proportional to its distance from the stationary layer and it will be maximum for the most distant layer, i.e. the top most layers. If a specific layer of a liquid is taken, then it is observed that a layer above it is moving faster and a layer immediately below is moving slower.

3.10.1 Coefficient of Viscosity:

Consider a layer AB of a liquid moving with a velocity v with respect to a parallel layer CD which is at a distance r from it see Figure (3.22).

Consider that the force required to produce the motion F acting on an area A and this force is acting along the direction AB , i.e. along the direction of motion . An equal force will, therefore, act on it in the backward direction due to viscosity .

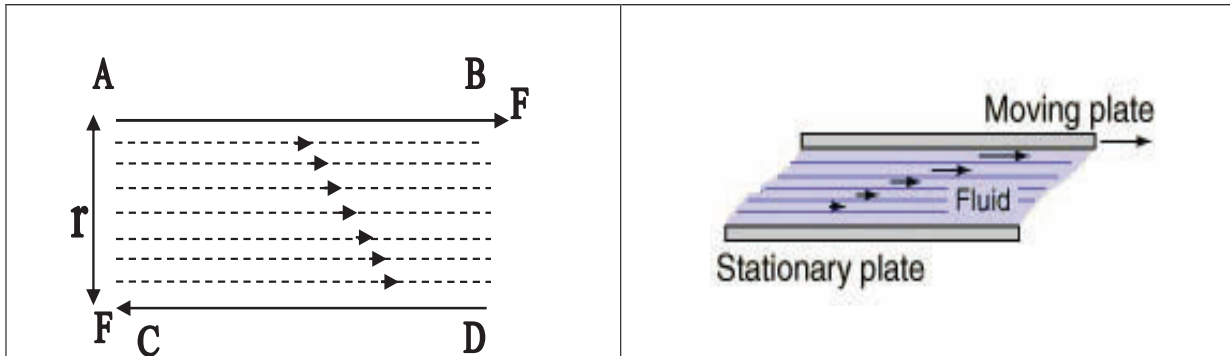


Figure (3.22)

The backward force F will depend on the following factors :

- 1- The relative velocity V , it is found that the magnitude of the force F is directly proportional to V and acts in the direction opposite to the direction of motion , i.e.

$$F \propto -V$$

- 2- The area on which F acts . It is found that the magnitude of F is directly proportional to A , i.e.

$$F \propto A$$

- 3- The distance r . It is found that the magnitude of F is inversely proportional to r , i.e.

$$F \propto 1/r$$

Then, it follows that

$$F \propto -AV/r$$

Or

$$F = -\eta AV/r \quad (3.13)$$

Where the constant of proportionality η is called the coefficient of viscosity and it depends on the nature of the fluid . The negative sign must be introduced because v decreases as r increases . If the two layers AB and CD are very close to each other , the relation : $F = -\eta AV/r$ can be written as :

$$F = -\eta A dV/dr \quad (3.14)$$

Where dv/dr is called the velocity gradient or rate of change of velocity with distance. If $A = 1 \text{ cm}^2$ and $dv/dr = 1$, then :

$$F = \eta \quad (\text{in magnitude})$$

So the coefficient of viscosity is defined as the tangential force per unit area required to maintain a unit velocity gradient, i.e. a unit velocity between two layers which are a unit distance apart.

From equation (3.14) the coefficient of viscosity η can be written as

$$\eta = - FV / Ar$$

$$\eta = - \left(\frac{F}{A} \right) / \left(\frac{V}{r} \right) \quad (3.15)$$

The quantity F/A is the shear stress exerted on the fluid and the quantity (v/r) is the rate of change of shear strain, therefore,

$$\eta = - \text{shear stress} / \text{rate of change of shear strain}$$

The SI unit of the coefficient of viscosity η is Nsm^{-2} , this unit is called Pascal second (Pa.sec) or Poiseuilles (PI). In cgs system the unit of η is (dyne second cm^{-2}) and it is the commonly used unit, and is called poise, where:

$$1 \text{ poise} = 1 \text{ dyne sec cm}^{-2}$$

$$= 10^{-1} \text{ N sec m}^{-2}$$

Or $1 \text{ Poiseuilles} = 10 \text{ poise}$

3.10.2 Poiseuille's Formula:

Consider a viscous liquid that flows in a cylindrical tube of length l and radius R such that:

- The flow of liquid is parallel to the axis of the tube.
- The flow is steady, i.e. no acceleration of the flow exists.
- The velocity of the liquid layer in contact with the walls is zero and increases regularly and continuously towards the inner side, it being maximum along the axis of the tube.

This flow of a viscous liquid is called the laminar, in which the velocity is greatest at the center of the tube and decreases to zero at the walls.

3.10.3 Stoke's Formula for The Velocity of A Small Sphere Falling Through A Viscous Liquid :

When a steel ball is dropped into a viscous liquid in a tall jar see Figure (3.23), it begins to move down with acceleration under gravitational pull. The motion of the ball in the liquid is opposed by viscous forces. These viscous forces increase as the velocity of the ball increases. Finally a velocity will be attained when the apparent weight of the ball becomes equal to the retarding viscous forces acting on it. At this

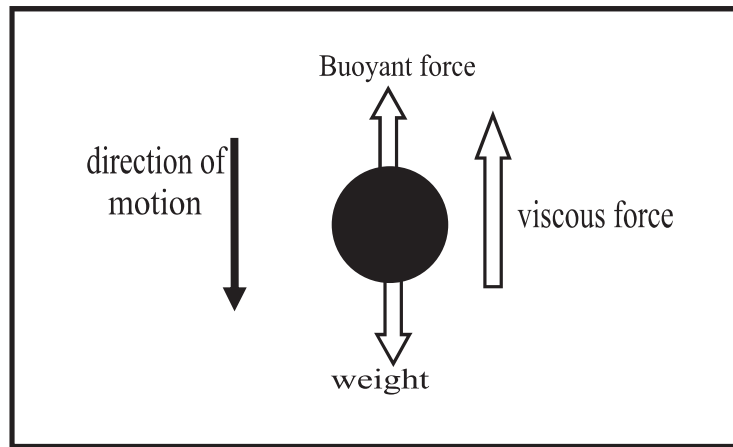


Figure (3.23)

stage , the resultant force on the ball is zero . Therefore the ball continues to move down with the same velocity thereafter . This uniform velocity is called terminal velocity .

For a small sphere falling through a viscous fluid , the opposing force is depends on :

- The terminal velocity v of the ball
- The radius r of the sphere
- The coefficient of viscosity η

Combining all these factors , we have

$$F \propto V^a \eta^b r^c$$

Or
$$F = K V^a \eta^b r^c$$

Where K is dimensionless constant . Stoke found experimentally that :

$$F = 6 \pi V r \eta \quad (3.6)$$

As shown in Figure (3.22), if ρ is the density of the ball and ρ' is the density of the liquid , then :

- The downward force due to gravity (weight of the ball) ,

$$W = \left(\frac{4}{3} \pi \right) r^3 \rho g$$

- The upward force (buoyant force) on the ball,

$$F_b = \text{weight of the displaced liquid} = \left(\frac{4}{3} \pi \right) r^3 \rho' g$$

- The viscous force, $F = 6 \pi V r \eta$

When the ball attains its terminal velocity, the resultant force on the ball is zero, therefore:

$$W = F_b + F$$

$$\left(\frac{4}{3}\pi\right)r^3\rho g = \left(\frac{4}{3}\pi\right)r^3\rho'g + 6\pi\eta r v \quad \text{or} \quad 6\pi\eta r v = \left(\frac{4}{3}\pi\right)r^3(\rho - \rho')g$$

Then
$$v = \left(\frac{2}{9}\right)\frac{r^2}{\eta}(\rho - \rho')g \quad (3.17)$$

3.11 Surface Tension

Surface tension is the energy required to increase the surface area of a liquid by a unit amount.

- ❖ For example, the surface tension of water at 20°C is $7.29 \times 10^{-2} \text{ J/m}^2$, which means that an energy of $7.29 \times 10^{-2} \text{ J}$ must be supplied to increase the surface area of a given amount of water by 1 m^2 . Water has a high surface tension because of its strong hydrogen bonds. The surface tension of mercury is even higher ($4.6 \times 10^{-1} \text{ J/m}^2$) because of even stronger metallic bonds between the atoms of mercury see Figure (3.24)

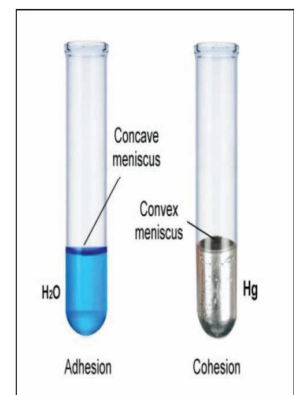


Figure (3.24)

- ❖ **The surface tension** is defined as the ratio of the magnitude of the surface tension force to the length along which the force acts:

$$\gamma = \frac{F}{L} \quad \text{N/m} \quad (SI \text{ unit}) \quad (3.18)$$

3.11.1 Measurement of Surface Tension:

Capillary rise method:

- if a capillary tube is immersed in a liquid such as water contained in beaker, the liquid immediately rises up the tube to a certain height, this rise of liquid in the tube occurs because the force of adhesion (forces between like molecules) between water molecule and the capillary wall is greater than the force of cohesion (forces between unlike molecules) between water molecules.
- The shape of the surface depends upon the relative size of the cohesive and adhesive forces:
 1. if adhesion > cohesion, the liquid clings to the walls of the container and the meniscus is hemispherically concave see Figure (3.25).

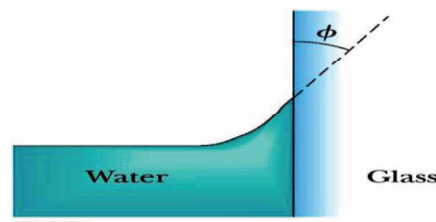


Figure (3.25)

2. if cohesion > adhesion, the forces cause decrease of the liquid level in the capillary below that in the chamber and the meniscus is hemispherically convex.
3. By measuring the rise in capillary, we have two forces opposing each other, the up word force (F_1) due to the surface tension and the counteracting force (F_2) due to the weight of the column of liquid in the tube see Figure (3.26).

$$F_1 = 2\pi r \gamma \cos \theta \quad (3.19)$$

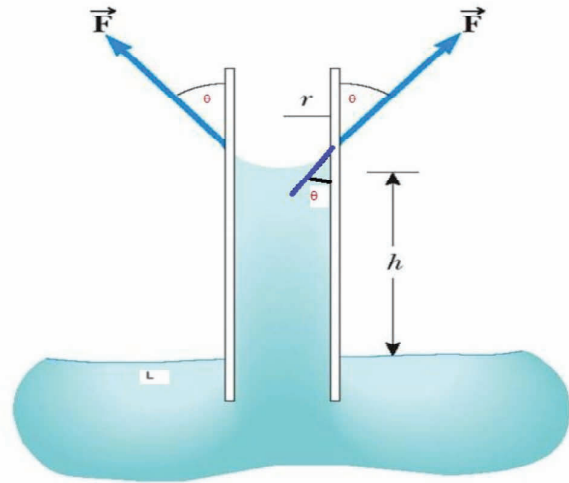


Figure (3.26)

Where θ : is the angle between surface of liquid and capillary wall, r : is the inside radius of the capillary tube, γ : is the liquid surface tension.

- When the liquid such as water wets the surface of the capillary tube, the θ is taken as unity, then $F_1 = 2\pi r \gamma$ (3.20)
- the counteracting force $F_2 = \pi r^2 \rho g h = \text{pressure at point} \times \text{area}$ (3.21)

Where h : is the height of liquid column in the capillary tube up to the lowest point of the meniscus, ρ : is the density of liquid, g : is the acceleration of gravity.

- At equilibrium the up word force (F_1) is equal to the down word force (F_2), the liquid does not move in the capillary:

$$F_1 = F_2 \quad \text{i.e.} \quad (3.20) = (3.21)$$

$$2\pi r \gamma = \pi r^2 \rho g h \Rightarrow \gamma = \frac{\pi r^2 \rho g h}{2\pi r}$$

$$\gamma = \frac{1}{2} r \rho g h \quad (\text{N/m}) \quad (3.22)$$

Example (3.11)

Given that the surface tension of ethanol is (0.032 J/m^2). calculate the capillary rise in a glass tube that is (0.1 mm) in radius. Assumed that, the density of ethanol is (0.71 g/cm^3).

Solution

$$\text{surface tension is } \gamma = \frac{1}{2} r \rho g h \quad \text{N/m} \Rightarrow$$

$$h = \frac{2\gamma}{r \rho g} = \frac{2 * 0.032}{710 * 9.8 * 0.1 \times 10^{-3}} = 0.91 \text{ m}$$

3.12. Problems**Choose The Correct Answer in Each of The Followings:**

- 1) A fluid of density $9.1 \times 10^2 \text{ kg/m}^3$ is flowing through a tube at a speed of 5.3 m/s. What is the kinetic energy density of the fluid?
- A) Cannot be calculated without knowing the pressure
 - B) Cannot be calculated without knowing the elevation
 - C) $4.8 \times 10^3 \text{ J/m}^3$
 - D) $1.3 \times 10^4 \text{ J/m}^3$
- 2) The equation of continuity for fluid flow can be derived from the conservation of:
- A) Energy
 - B) Mass
 - C) Angular momentum
 - D) Volume
- 3) A change in the pressure applied to a fluid is transmitted undiminished to every point of the fluid and to the walls of the container.
- A) Strain
 - B) Archimedes' principle
 - C) Pascal's law
 - D) Pressure
- 4) All fluids are:
- A) gases
 - B) liquids
 - C) gases or liquids
 - D) non-metallic
- 5) Bernoulli's equation can be derived from the conservation of:
- A) Energy
 - B) Mass
 - C) Angular momentum
 - D) Volume
- 6) A long U-tube contains mercury (Density = $14 \times 10^3 \text{ kg/m}^3$). When 10 cm of water (density = 1000 kg/m^3) is poured into the left arm, the mercury in the right arm rises above its original level by (in cm):
- A) 0.36
 - B) 0.71
 - C) 14
 - D) 35

7) The magnitude of the buoyant force on an object always equals the weight of the fluid displaced by the object.

- A) Strain
- B) Archimedes' principle
- C) Pascal's law
- D) Pressure

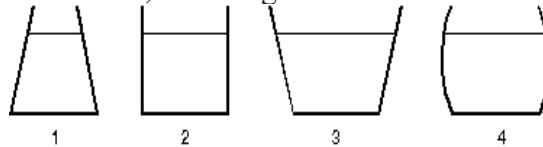
8) Barometers and open-tube manometers are two instruments that are used to measure pressure.

- A) Both measure gauge pressure
- B) Both measure absolute pressure
- C) Barometers measure gauge pressure and manometers measure absolute pressure
- D) Barometers measure absolute pressure and manometers measure gauge pressure

9) The vessels shown below all contain water to the same height. Rank them according to the pressure exerted by the water on the vessel bottoms, least to greatest.

- A) 1, 2, 3, 4
- B) 3, 4, 2, 1
- C) 1, 2, 4, 3

D All pressures are the same



10) 100 kg body is standing on a square surface that's length is 10 cm then the pressure(in Pa) is:

- A) 1.2×10^{-7}
- B) 1.2×10^{-5}
- C) $9.8 \times 10^{+4}$
- D) $9.8 \times 10^{+6}$

11) A rock, which weighs 1400 N in air, has an apparent weight of 900 N when submerged in fresh water (998 kg/m^3). The volume of the rock(in m^3) is:

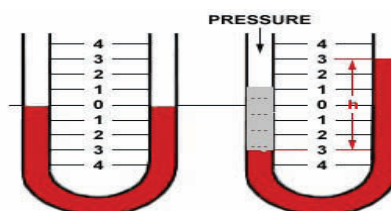
- A) 0.14
- B) 0.50
- C) 0.90
- D) 5.1×10^{-2}

12) An object hangs from a spring balance. The balance indicates 30N in air and 20N when the object is submerged in water. What does the balance indicate (in N) when the object is submersed in a liquid with a density that is half that of water?

- A) 20
- B) 25
- C) 30
- D) 35

13) Assume the height of water in the manometer is at 0 level in both tube sides, then a liquid is poured inside the left side, if the density of water is 1000 kg/m^3 , then the density of this liquid equals (In kg/m^3)

- A) 1500
- B) 1000
- C) 500
- D) 2500



14) Suppose the atmospheric pressure equals $1 \times 10^5 \text{ Pa}$, find the pressure (in Pa) at a height of 1.5 km if the density of air is 1.21 kg m^{-3}

- A) 1.2×10^{-7}
- B) 1.2×10^{-5}
- C) 8.2213×10^4
- D) 18956.2

15) A wood board floats in fresh water with 60% of its volume under water. The density of the wood is: (if the density of water is 1 g/cm^3)

- A) 0.4 g/cm^3
- B) 0.5 g/cm^3
- C) 0.6 g/cm^3
- D) less than 0.4 g/cm^3

16) A plastic sphere floats in glycerin with 40.0% of its volume submerged, then the density of the glycerin (in kg/m^3) is: (If the density of sphere equals 500 kg/m^3)

- A) 1500
- B) 1000
- C) 1250
- D) 400

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Chapter (4)

Temperature, Heat, and the First Law of Thermodynamics



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Chapter (4)

Temperature, Heat, and the First Law of Thermodynamics

Content

Thermal Basics: Temperature - Temperature Scales and Thermometers. Thermal Concept: Units of thermal energy - Heat capacity and Specific Heat - Newton's law of cooling - Specific heat of gases. Heat Transfer: Thermal conduction - Prevost's theory of heat change - Energy distribution of Black body radiation - Stefan's law- Thermal Expansion -Latent Heat.

Learning Objective

1. Identify the lowest temperature as 0 on the Kelvin scale (absolute zero).
2. Explain the zeroth law of thermodynamics.
3. Convert a temperature between any two (linear) temperature scales, including the Celsius, Fahrenheit, and Kelvin scales.
4. Identify that a change of one degree is the same on the Celsius and Kelvin scales.
5. Know the thermal expansion, quantity of heat and latent Heat.
6. Identify Heat Transfer and Stefan's law.

4.1 Temperature

The temperature of a body is its degree of hotness (or coldness). Thus, temperature is a measure of how hot (or cold) a body is, and should not be confused with the amount of heat it contains. i.e. temperature define as

It is the average energy of the object's molecules.

*Temperature: It is used to specify how hot or cold an object feels.

**Temperature depends on the transnational molecular motion.

Heat

It is the total energy of the object.

OR, Transfer energy from one body to another due to the difference in temperature.

Thermal Equilibrium

Two systems are in thermal equilibrium if and only if the are at the same temperature.

Zeroth Law of Thermodynamics

“When each of two systems is in thermal equilibrium with a third, the first two systems must be in thermal equilibrium with each other” see figure (4.1)

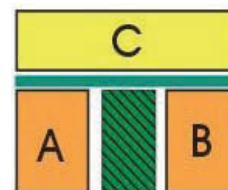


Figure (4.1)

Thermometer

It is a device used to measure the temperature of the object.

*Given short notes about the basic physical properties used for making thermometer?

Some of the properties used to measure temperature:

1. The change in volume of a liquid
2. The change in length of solid
3. The change in volume of a gas at constant pressure
4. The change in pressure of a gas at constant volume
5. The change in electric resistance of a conductor
6. The change in color of a very hot body

4.2 Temperature Scales

There are many types of thermometers, but each makes use of a particular thermometric physical property (i.e. a property whose value changes with temperature T). For example: mercury in glass thermometer makes use of the change in length (l) of a column of mercury confined in the capillary tube of uniform bore ($l \propto T$), a platinum resistance thermometer makes use of the increase in electrical resistance with increasing temperature ($R \propto T$),

In order to establish a temperature scale it is necessary to make use of **fixed points**: a fixed point is a single temperature at which certain physical property always occurs. Three such points are defined below.

The ice point (Lower fixed point):	Is "the temperature at which pure ice can exist in equilibrium with pure water at standard atmospheric pressure".
The Steam point (Upper fixed point):	Is "the temperature at which pure water can exist in equilibrium with pure water vapor at standard atmospheric pressure".
Fundamental intervals:	is "the interval between the lower and upper fixed points on the temperature scale".

Triple point: Is "unique temperature at which ice, pure water and pure water vapor can exist together in equilibrium".

The triple point is particularly useful, since there is only one pressure at which all three phases (solid, liquid and gas) can be in equilibrium with each other.

The SI unit of temperature is the Kelvin (K). An interval of one Kelvin is defined $1/273.15$ of the temperature of the triple point of water as measured on thermodynamic scale of temperature.

- Another unit, the degree Celsius ($^{\circ}\text{C}$), is often used and defined by

$$T_c = T_K - 273.15 \quad (4.1)$$

Where T_c = temperature in $^{\circ}\text{C}$, and T_K = temperature in K.

The temperature scales are: Celsius, Fahrenheit, and Kelvin see figure (4.2).

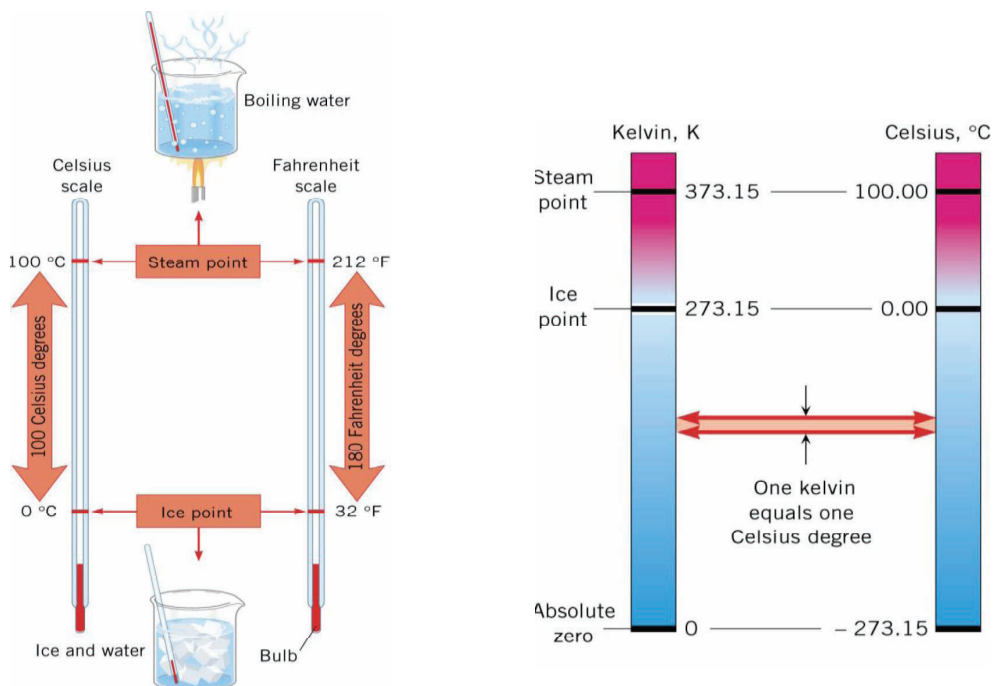


Figure (4.2)

Celsius (°C)	Fahrenheit (°F)	Kelvin (K)
Steam point 100	Steam point 212	Steam point 373.15
Ice point 0	Ice point 32	Ice point 273.15
$T_C = \frac{5}{9}(T_F - 32)$ $T_C = T_K - 273.15$	$T_F = \frac{9}{5}(T_C) + 32$	$T_C = T_K - 273.15$

We can deduce the above equations from this equation:

$$\frac{T_C - 0}{100} = \frac{T_F - 32}{180} = \frac{T_K - 273.15}{100}$$

Example (4.1)

You place a small piece of melting ice in your mouth. Eventually, the water all converts from ice at $T_1 = 32$ °F to body temperature, $T_2 = 98.6$ °F. Express these temperatures as °C and K, and find $\Delta T = T_2 - T_1$, in both cases?

$$T_C = \frac{5}{9}(T_F - 32) = \frac{5}{9}(98.6 - 32) = 37^\circ C$$

$$T_K = T_C + 273.15 = 37^\circ C + 273.15 = 310.15 K$$

$$\Delta T_K = \Delta T_C = 37$$

Example (4.2)

The extremes of temperature in the bottom of the earth, over a period of 50 years, differ by 116 °F. Express this range in Celsius degree?

$$\Delta T_C = \frac{5}{9}(\Delta T_F) = \frac{5}{9}(116) = 64.44^\circ C$$

Example (4.3) : Conversions between Temperature Scales

What is 0 K on (a) the Celsius scale and (b) the Fahrenheit scale? (c) What is a room temperature of 72°F on the Celsius scale?

Solution

$$(a) \quad \because T_K = T_C + 273.15 \Rightarrow T_C = T_K - 273.15 = 0 - 273.15 = -273.15^\circ C$$

$$(b) \quad \because T_F = \left(\frac{9}{5}T_C\right) + 32 \Rightarrow T_F = \frac{9}{5}(-273.15) + 32 = -459.67^\circ F$$

$$(c) \quad \because T_F = \frac{9}{5}T_C + 32 \Rightarrow T_C = \frac{5}{9}(T_F - 32) = \frac{5}{9}(72 - 32) = 22^\circ C$$

Example (4.4)

(a) The normal temperature of the human body is 98.6 °F. What is it on the Celsius scale? On the Kelvin scale?

(b) If the air temperature is -15 °C, what is it in degrees Fahrenheit?

(c) What is a temperature change of 20 °C expressed in Kelvin? In degrees Fahrenheit?

Solution

$$(a) \quad \because T_F = \frac{9}{5}T_C + 32 \Rightarrow T_C = \frac{5}{9}(T_F - 32) = \frac{5}{9}(98.6 - 32) = 37^\circ C$$

$$\text{And } \because T_K = T_C + 273.15 \Rightarrow T_K = 37 + 273.15 = 310.15 K$$

i.e. the temperature of human body is 37 °C or 310.15 K

$$(b) \because T_F = \frac{9}{5}T_C + 32 \Rightarrow T_F = \frac{9}{5}(-15) + 32 = 5^\circ F$$

$$(c) \because T_F = \frac{9}{5}T_C + 32 \Rightarrow \Delta T_F = \frac{9}{5}(\Delta T_C) \text{ and } \because T_K = T_C + 273 \Rightarrow \Delta T_K = \Delta T_C$$

Then , the temperature change of 20 °C in Kelvin is also 20 K and

$$\Delta T_F = \frac{9}{5}(\Delta T_C) = \frac{9}{5}(20) = 36^\circ F$$

Constant Volume Gas Thermometer

- The used property is the change in pressure of a gas at constant volume as the temperature changes.
- When the gas is heated (or cooled), its pressure increase (decrease), the volume of the gas in bulb (B) is kept constant, by raising or lowering the reservoir (R) see figure (4.3).
- At low pressure, The gases behave like ideal gases with relation,

$$PV = nRT = (\text{Constant}) T$$

-

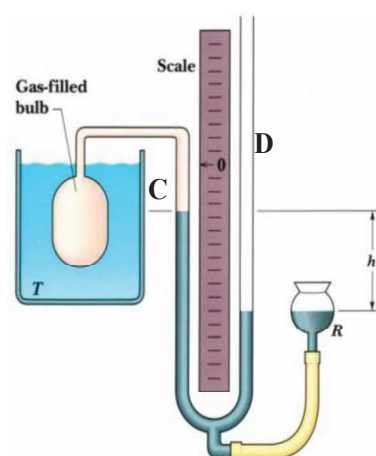


Figure (4.3)

Where: P: pressure , V : Volume , n: number of mole , R: Constant of gas ,T: temperature en kelvin .

- The triple point of water is the point at which water, ice and vapor coexist in equilibrium see figure (4.4).

- **Kelvin unit:** is the fraction $\frac{1}{273.16}$ of the thermodynamic temperature of the triple point of water.

- **The relation is:** $T = 273.16 \frac{P}{P_3}$

Where,

P_3 : is the pressure at triple point

T : is the gas temperature in Kelvin

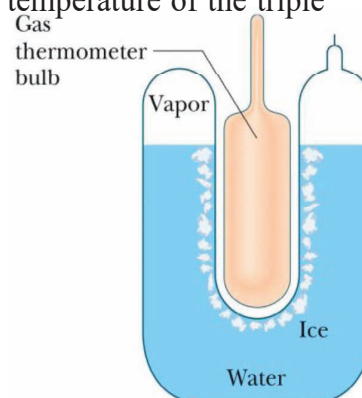


Figure (4.4)

Triple point of the water: " is the temperature and pressure at which ice, water and steam (S,L,G) can all present " and occurs at $\Delta T = 0.01^\circ\text{C}$ and $P = 0.006\text{ atm}$

$$\therefore P = \text{const} \times T \Rightarrow P_{\text{triple}} = \text{const} \times T_{\text{triple}}$$

$$\therefore \frac{P}{P_{\text{triple}}} = \frac{T}{T_{\text{triple}}}$$

Exercise

$T^\circ\text{C}$	$T^\circ\text{F}$	T_{K}
.....		273.15
.....		
.....	68	
- 40		
.....		298.15

4.3 Thermal Expansion of Solids and Liquids

4.3.1 Expansion of Solids

With few exceptions, substances expand when heated, and very large forces may be set up if there is an obstruction to the free movement of the expanding or contracting bodies. If concrete road surfaces were laid down in one continuous piece cracks would appear owing to expansion and contraction brought about by the difference between summer and winter temperatures. To avoid this, the surface is laid in small sections, one being separated from the next by a small gap which is filled in with a compound of pitch. On a hot summer day, expansion often squeezes this material out of the joints see figure (4.5).

In the older methods of laying railway tracks gaps have to be left between successive lengths of rail to allow for expansion. Even when such gaps have been left

The rails may sometimes "creep" and close up the gaps. If this happens a rise in temperature may lead to buckling of the track see Figure (4.5). Free movement at the rail joints is allowed for by making the bolt holes slotted as shown Figure (4.5).

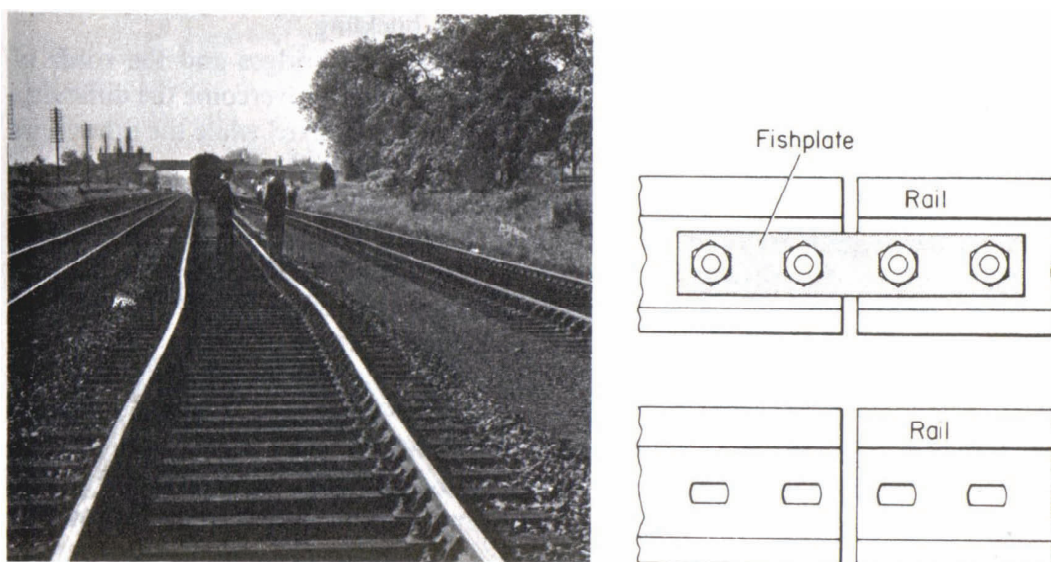


Figure (4.5) Railway lines distorted by expansion during hot weather.

In modern practice, however, railway lines are welded together to form long, continuous lengths. With this method, it is only the last fifty to one hundred meters at any length which show expansion, usually of a few centimeters. This movement is taken up by planning the ends of the rails and overlapping them. The remainder of the rails are unable to expand and so the forces set up develop internal potential energy in the metal. To keep this internal energy to a minimum, it is best to lay the track at a time when the temperature is midway between the summer and winter averages. This technique has been made possible by the use of concrete sleepers and improved methods of fixing the tails so that the track may withstand the thermal stresses set up in it without buckling.

Allowance also has to be made for the expansion of bridges and the roofs of buildings made of steel girders. Various methods are used to overcome the difficulty, a common one being to have one end only of the structure fixed while the other rests on rollers. Free movement is thus permitted in both directions.

Over a very long period of years, expansion and contraction causes "creeping" of lead on the sloping roofs of buildings when heated by the sun, the lead expands and tends to move down the roof under the force of gravity. On cooling and contracting, the force of contraction is opposed by the force of gravity on the lead and friction between it and the roof planking. This sets up a strain in the lead and gives it a very slight permanent stretch. After many years the lead stretches more and more, and eventually it forms into folds and may even break.

This trouble has been aggravated by the all too common practice in the past of using lead in very large sheets. When restorations are carried out it is now usual to replace the lead in much smaller sections.

4.3.2. Useful Applications of Thermal Expansion

4.3.2.1. Linear Expansion

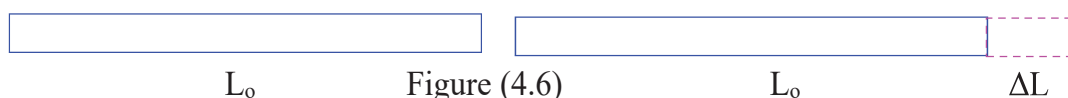
- Suppose a rod of material, Figure (4.6), has a length L_0 at some initial temperature T_0 when the temperature changes by ΔT , the length changes by ΔL .
- The length change ΔL depends on:
 - The original length L_0
 - The temperature change ΔT
 - The kind of material

i.e., the length change is proportional to L_0 and ΔT , so the proportional constant depends on the kind of the material (naming **the coefficient of linear expansion α**)

$$\Delta L = \alpha L_0 \Delta T$$

So, the length L at a temperature ($T = T_0 + \Delta T$) is:

$$L = L_0 + \Delta L = L_0 + \alpha L_0 \Delta T = L_0 (1 + \alpha \Delta T)$$



The fractional length change = $\Delta L / L_0$.

Example (4.5)

The concrete sections ($\alpha = 12 \times 10^{-6} \text{ } ^\circ\text{C}^{-1}$) are designed to have a length of 10.0 m. The sections are poured and cured at 20°C . What minimum spacing should the engineer leave between the sections to eliminate buckling if the concrete is to reach a temperature of 50°C ?

Answer

$$\Delta L = \alpha L_0 \Delta T = \alpha L_0 (T_f - T_i) = 12 \times 10^{-6} \text{ } ^\circ\text{C}^{-1} \times 10 \times (50 - 20) = 3.6 \times 10^{-3} \text{ m}$$

Example (4.6)

A surveyor uses a steel measuring tape that is exactly 50m long at a temperature of 20°C . What is the length at a temperature 35°C . If the measured distance at that temperature is 35.794m, what is the actual distance?

where $\alpha = 1.2 \times 10^{-5} \text{ K}^{-1}$.

Answer

$$L = L_0 (1 + \alpha \Delta T)$$

$$L = 50 [1 + 1.2 \times 10^{-5} \times (50 - 35)] = 50 [1 + 18 \times 10^{-5}] = 50 \times 1.00018 = 50.009 \text{ m}$$

$$L = L_0 (1 + \alpha \Delta T)$$

$$35.794 = L_0 [1 + 1.2 \times 10^{-5} \times (35 - 20)] = L_0 [1 + 18 \times 10^{-5}] = L_0 \times 1.00018$$

$$\rightarrow L_0 = 35.7876 \text{ m}$$

Example (4.7)

A steel measuring tape measure the actual distance at 20 °C. The measured distance is 35.794m at the temperature 35 °C. what is the actual distance ? (note that $\alpha = 1.2 \times 10^{-5} / ^\circ\text{C}$)

Answer

$$L = L_0(1 + \alpha \Delta T)$$

$$L = 35.794 \left[1 + 1.2 \times 10^{-5} (20 - 35) \right] = 35.794 \times 0.99982$$

$$L = 35.7876 \text{ m}$$

4.3.2.2. Area Expansion

1. Suppose a rectangular face of a block, Figure (4.7), has a length of L_0 and width of W_0 at some initial temperature T_0 .
2. If the material has a linear expansion coefficient α and the temperature is changed by ΔT then:

- The new length is $L = L_0(1 + \alpha \Delta T)$
- The new width is $W = W_0(1 + \alpha \Delta T)$
- The original area is $A_0 = L_0 W_0$

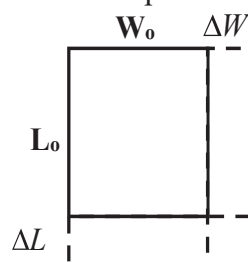


Figure (4.7)

↓ The new area is $A = L W = L_0(1 + \alpha \Delta T) \times W_0(1 + \alpha \Delta T)$

$$A = L_0 W_0 (1 + \alpha \Delta T)^2 = L_0 W_0 (1 + 2\alpha \Delta T + \alpha^2 \Delta T^2) \approx A_0 (1 + 2\alpha \Delta T)$$

Because $\alpha^2 \Delta T^2 \approx 0$

$$A \approx A_0 (1 + \beta \Delta T) \quad \text{where } \Delta A = A_0 \beta \Delta T, \quad \beta = 2\alpha$$

*if the temperature of a disk with a hole has changed (increase or decrease) then the dimension of the disk and the hole is changed (increase or decrease).

Example (4.8)

A brass disk has a hole 80 mm in diameter punched in its center at 70 °F. If the disk is placed in a boiling water, what will be the new area of the hole?

($\alpha = 1 \times 10^{-5} / ^\circ\text{F}$).

Answer

$$A_0 = \pi r^2 = 3.14 \times (80/2)^2 = 5026.55 \text{ mm}^2$$

$$\beta = 2 \times \alpha = 2 \times 1 \times 10^{-5} = 2 \times 10^{-5} / ^\circ\text{F}$$

$$A = A_0 (1 + \beta \Delta T) = 5027 \times [1 + 2 \times 10^{-5} \times (212 - 70)]$$

$$A = 5026.55 \times 1.00284 = 5040.72 \text{ mm}^2$$

4.3.2.3. Volume Expansion

1. Temperature increases usually cause increases in volume for both solid and liquid materials See Figure (4.8).
2. The increase in volume ΔV is proportional to the temperature change and the initial volume V_0 .

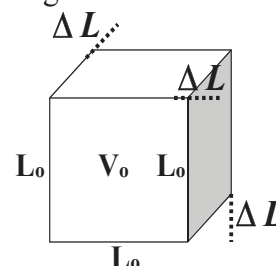


Figure (4.8)

$$V = L^3 = [L_0(1 + \alpha \Delta T)]^3 = L_0^3(1 + \alpha \Delta T)^3 = V_0[1 + (\alpha \Delta T)]^3$$

$$V = V_0[1 + 3(\alpha \Delta T) + 3(\alpha \Delta T)^2 + (\alpha \Delta T)^3] \approx V_0[1 + 3\alpha \Delta T]$$

Because: $3\alpha^2 \Delta T^2 \approx 0$, $3\alpha^3 \Delta T^3 \approx 0$

$$V \approx V_0(1 + \gamma \Delta T) \quad \text{where } \Delta V = V_0 \gamma \Delta T, \quad \gamma = 3\alpha$$

Example (4.9)

A solid block of steel has dimensions 200 mm x 50 mm x 40 mm at 20 °C. Its density at 20 °C is 7800 kg/m³.

calculate: (a) the mass in kilogram; (b) the increase in volume, in mm³, if the temperature rises to 220 °C. (c) the density at 220 °C.

Take $\alpha = 11 \times 10^{-6} / ^\circ\text{C}$.

Answer

$$V_0 = 200 \times 50 \times 40 = 4 \times 10^5 \text{ mm}^3 = 4 \times 10^5 \times 10^{-9} \text{ m}^3 = 4 \times 10^{-4} \text{ m}^3$$

$$\text{Mass} = \text{density} \times \text{Volume} = 7800 \times 4 \times 10^{-4} = 3.12 \text{ Kg}$$

$$\Delta V = V_0 \gamma \Delta T = 3\alpha V_0 \Delta T = 3 \times 11 \times 10^{-6} \times 4 \times 10^5 \times (220 - 20) = 2640 \text{ mm}^3$$

$$\text{Density} = \text{mass} / \text{Volume} = 3.12 / (402640 \times 10^{-9}) = 7750 \text{ kg} / \text{m}^3$$

4.3.3. Expansion Of Liquids

We have already seen that liquids expand, in connection with our study of liquid-in glass thermometers. The expansion of a liquid may be shown by means of a flask filled with a rubber bung and a length of glass tubing Figure (4.9). The flask is filled with water or other liquid and the bung pushed in until the level of the liquid comes a short distance up the tube. On plunging the flask into a can of hot water it is noticed that the level of the liquid at first falls slightly and then starts to rise steadily.

The initial fall in level is caused by the expansion of the glass which becomes heated and expands before the heat has had time to be conducted through the glass into the liquid.

It is known that different liquids have different thermal expansions. To demonstrate this, several fairly large glass bulbs with glass stems are filled to a short distance above the bulb With various liquids Figure (4.9).

In order to make a fair comparison, the bulbs and steins must all be of the same size. The bulbs are immersed in a metal trough containing cold water and left until they have reached a steady temperature. A little extra liquid should now be added, where necessary, to make all levels the same. The bath is now heated and well stirred to ensure a uniform temperature. When the bulbs and their contents have acquired the new temperature of the bath it will be seen that the liquid levels have risen by different amounts. Thus, for a given rise in temperature, equal volumes of different liquids show different expansions in volume

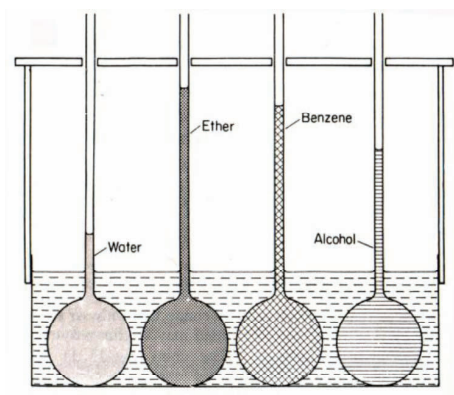


Figure (4.9) Comparison of expansion

Example (4.10)

On a hot day in Las Vegas, oil trucker loaded **37000L** of diesel fuel. He encountered cold weather on the way to Payson, Utah, where the temperature was **23K lower** than in Las Vegas, and where he delivered his entire load. How many liters did he deliver? ($\beta_{\text{diesel}} = 9.5 \times 10^{-4} \text{ }^\circ\text{C}^{-1}$).and neglect the steel tank expansion.

Solution:

$$V_{o(\text{diesel})} = 37000L ; \Delta T = -23K ; V_{\text{diesel}} = ?$$

$$\Delta V = V_o \beta \Delta T = (37000L)(9.5 \times 10^{-4} \text{ }^\circ\text{C}^{-1})(-23K) = -808L$$

$$V = V_o + \Delta V = 37000L - 808L = 36190L .$$

4.3.3.1 Real and Apparent Expansion of A Liquid

Unlike solids, liquids have no fixed length or surface area but always take up the shape of (lie containing vessel. Therefore, in the case of liquids we are concerned only with volume changes when they are heated.

The real (or absolute) expansivity of a liquid is the fraction of its volume by which it expands per Kelvin rise in temperature.

Any attempt at direct measurement of the expansion of a liquid is complicated by the fact that the containing vessel itself expands. However, since liquids must always be kept in

some kind of vessel, it is just as useful to know the apparent expansion of a liquid, which is the difference between its real expansion and the expansion of the vessel.

The apparent expansivity of a liquid is the fraction of its volume by which the liquid appears to expand per Kelvin rise in temperature when heated in an expansible vessel.

In this respect we shall not be concerned with methods of measuring these expansivities.

4.3.3.2 The Unusual Expansion of Water

Some substances do not always expand when heated. Over certain temperature ranges they contract. Water is an outstanding example see figure (4.10).

If we start with some ice at -10°C and supply it with heat, it expands just like any other solid until it reaches 0°C . After this it begins to melt while the temperature remains constant at 0°C . This inciting is accompanied by a contraction in volume of about 8 per cent. Between 0 and 4°C the water contracts still further, reaching its minimum volume at about 4°C . This means that water has a maximum density at 4°C .

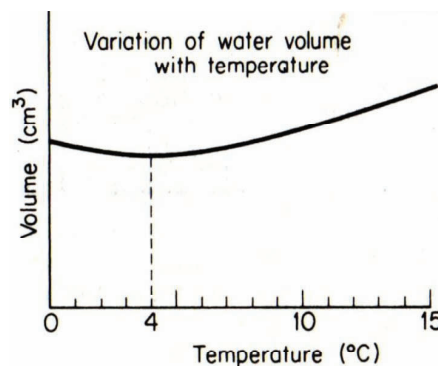


Figure (4.10)

Beyond 4°C the water expands. This behavior is described as anomalous (= irregular).

The changes in the water volume between 0 and 5°C are shown graphically in the above Figure. Unfortunately, on the scale of this graph, we cannot show the contraction in volume when ice melts, since this is nearly 700 times greater than the contraction "of water between 0 and 4°C ".

Incidentally, the contraction in volume when ice melts is matched by a corresponding expansion when water freezes to form ice. This explains why pipes burst during frosty weather though the damage does not become apparent.

4.3.3.2.1. Frost Heave

One problem brought about by the expansion of water on freezing has occurred in cold store buildings. This is frost heave, the name given to the damage caused to the buildings when the water in the subsoil beneath the site freezes and expands, causing upward bulging of the floor and damage to foundations and walls. This reached serious proportions about the mid-fifties as storage conditions went lower in temperature.

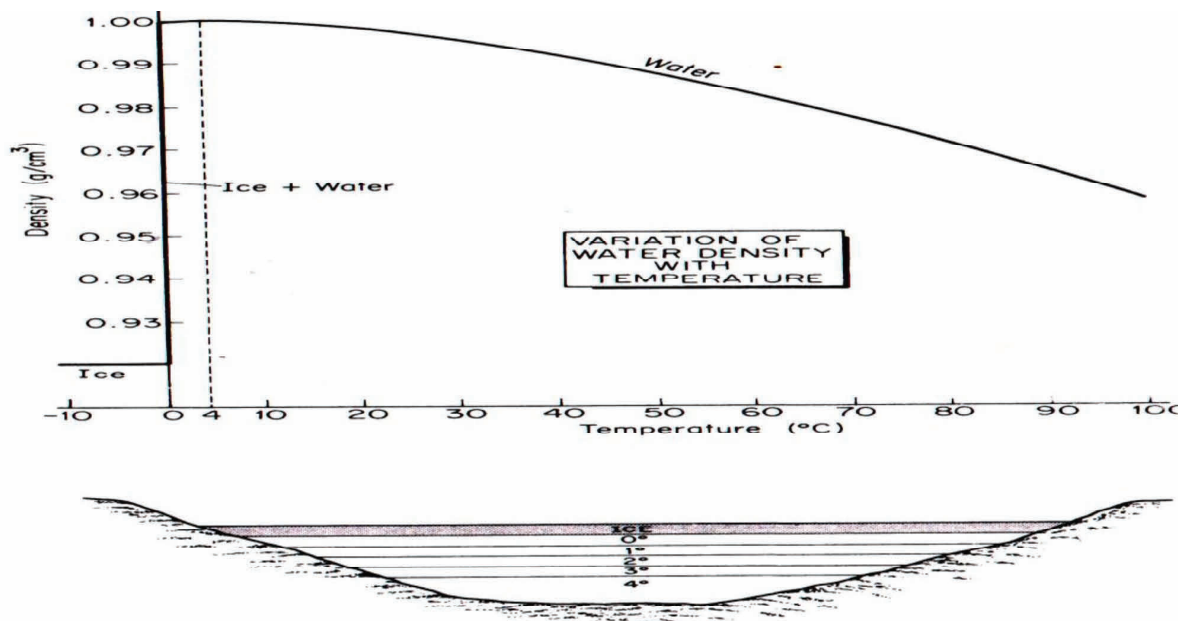
4.3.3.2.2. Density Changes in Water

The density changes which occur when a piece of ice at -10°C is gradually heated up to 100°C are shown in Figure(4.11). The maximum density region on this graph should be compared with the corresponding volume changes shown in Fig. 3.8.

Note how much greater is the density change when ice melts to form water at 0°C compared with the density change of water between 0°C and 4°C .

4.3.3.2.3. Biological Importance of The Anomalous Expansion of Water.

The peculiar expansion of water has an important bearing on the preservation of aquatic life during very cold weather Figure (4.11). As the temperature of a pond or lake falls, the water contracts, becomes denser and sinks. A circulation is thus set up until all the water reaches its maximum density at 4°C . If further cooling occurs any water below 4°C will stay at the top owing to its lighter density. In due course, ice forms on (the top of) the water, and after this (the lower layers of water at 4°C can lose heat only by conduction. Only very shallow water is liable to freeze solid. In deeper water there will always be water beneath (the ice in which fish and other creatures can live.



Thermal expansion of water

Figure (4.11)



Checkpoint 2

The figure here shows four rectangular metal plates, with sides of L , $2L$, or $3L$. They are all made of the same material, and their temperature is to be increased by the same amount. Rank the plates according to the expected increase in (a) their vertical heights and (b) their areas, greatest first.

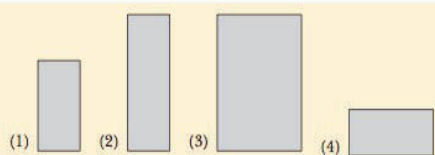


Figure (4.12)

Answer: (a) – 2 and 3 (same increase in height), then 1, and then 4
(b) – 3, then 2, then 1 and 4 (identical increase in area)

4.4. Quantity of Heat

- The quantity of heat is the thermal energy required to raise the temperature of a given mass.
- Thermal energy is the energy associated with random molecular motion.
- We can measure changes in thermal energy by relating it to change in temperature.
- Thermal energy lost or gained by objects is called heat.
- Internal energy (U) is the energy associated with the microscopic components of a system (i.e. the atoms and molecules).

4.4.1. Units of Quantity of heat:**4.4.1.1. Calorie (Cal):**

Is the quantity of heat required to change the temperature of 1 gram of water through 1°C?

4.4.1.2. British Thermal Unit (Btu) :

Is the quantity of heat required to change the temperature of 1 pound of water through 1°F.

$$\text{Btu} = 252 \text{ cal} = 0.252 \text{ Kcal}$$

4.4.1.3. Horsepower (hp)

It is the amount of power required to lift 33000 pound 1 ft in 1 minute, or 550 foot-pound per second.

$$\begin{aligned} 1 \text{ Btu} &= 252 \text{ Cal} \quad , \quad 1 \text{ hp} = 746 \text{ W} \quad , \\ 1 \text{ Cal} &= 4.186 \text{ J} \quad , \quad 1 \text{ food calorie} = 10^3 \text{ calorie} \end{aligned}$$

4.4.2. The Heat Capacity (of A Body):

Is the quantity of heat required to raise the temperature of the body one degree?

$$\text{Heat Capacity } S = \frac{Q}{\Delta T}$$

Its units: Cal/°C , Kcal/°C , J/K , Btu/°F

4.4.3. The Specific Heat Capacity

Is the quantity of heat required to raise the temperature of a unit mass of the material one degree?

$$C = \frac{Q}{m \Delta T} \quad \text{or} \quad Q = m C \Delta T$$

units: Cal/g.°C , Kcal/Kg.°C , J/Kg. K , Btu/lb_m.°F

4.4.4. Heat and Mechanical energy (Joule Experiment):

Joule carried an experiment shown in the figure (4.13) to give the relation between thermal energy units and mechanical energy units as shown:

- As the two blocks of mass m fall through a distance h , the loss in the potential energy (Which equal the work done by paddles) is given by :

$$W \propto Q$$

$$W = J \times Q$$

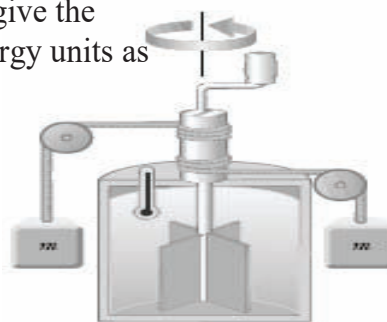


Figure (4.13)

J is the mechanical heat equivalent defined as the work done to produce a quantity of heat equal to 1 cal.

$$J = 4.186 \text{ Joule /cal}$$

From this we can conclude that

$$1 \text{ Btu} = 778 \text{ ft.lb}$$

$$1 \text{ cal} = 4.186 \text{ J}$$

$$1 \text{ Kcal} = 4186 \text{ J}$$

$$1 \text{ Btu} = 252 \text{ Cal} , \quad 1 \text{ lb}_m = 454 \text{ g} , \quad 1 \text{ Cal} = 4.186 \text{ J}$$

4.4.5. The Specific Heat (of A Material)(c)

It is the quantity of heat per unit mass required to raise the temperature by one degree Celsius (Kelvin). (a constant for each substance)

$$c = \frac{Q}{m \Delta T} \quad \text{Cal / g.}^\circ\text{C or Btu / lb.}^\circ\text{F or J / kg. K}$$

$$Q = m c \Delta T$$

***The heat lost by the warm bodies must equal the heat gained by the cool bodies.**

Example (4.11)

80 kg man ran a fever of 2°C above normal; whose temperature was 39°C instead of the normal 37°C . Assuming that the human body is mostly water, how much heat is required to raise his temperature by amount? $c_w = 4190 \text{ J / kg. K}$

Solution

$$\Delta T = 39 - 37 = 2^\circ\text{C} = 2 \text{ K}$$

$$\therefore Q = m c \Delta T = (80 \text{ Kg})(4190 \text{ J / Kg.K})(2 \text{ K}) = 6.7 \times 10^5 \text{ J} = 160 \text{ Kcal}$$

4.5. Heat Exchange

The term heat has now been introduced as the thermal energy absorbed or released during a temperature change. The principle of thermal equilibrium tells us that whenever object are place together in an insulated enclosure, they will eventually reach the same temperature. This is the result of a *transfer of thermal energy from the warmer bodies to the cooler bodies*. If energy is to be conserved, we say that heat lost by warm bodies must equal the heat gained by the cool bodies, That is heat lost = heat gained

$$\text{Total Heat Lost} = \text{Total Heat Gained}$$

4.5.1. Application Mixing Experimental

If hot body dropped into color container (**Calorimeter**), at the equilibrium temperature of the mixed system

$$\text{Total Heat Lost}(\text{hot body}) = \text{Total Heat Gained}(\text{cold body})$$

$$m_h c_h \Delta T_h = m_c c_c \Delta T_c \Rightarrow m_h c_h (T_h - T_{eq}) = m_c c_c (T_{eq} - T_c)$$

Example (4.12)

A 0.05 Kg unknown (ingot) of metal is heated to 200 °C and then dropped into a beaker containing 0.4 Kg of water initially at 20 °C. If the final equilibrium temperature of the mixed system is 22.4 °C. if the $c_w = 4190 \text{ J/kg} \cdot ^\circ\text{C}$

- Find the specific heat of the metal.
- What is the total heat transferred to the water.

Answer

The mass of the metal (m_s) = 0.05 kg, & $T_s = 200^\circ\text{C}$,

The mass of water (m_w) = 0.4 kg, & $T_w = 20^\circ\text{C}$, $T_f = 22.4^\circ\text{C}$

- Find the specific heat of the metal.

$$\text{Heat lost} = \text{Heat gained}$$

$$m_s c_s \Delta T_s = m_w c_w \Delta T_w$$

$$0.05 \times c_s \times (200 - 22.4) = 0.4 \times 4190 \times (22.4 - 20)$$

$$c_s = 453 \text{ J/kg} \cdot ^\circ\text{C}$$

- What is the total heat transferred to the water.

$$\text{The total transferred heat} = m_w c_w \Delta T_w = 0.4 \times 4190 \times 2.4 = 4022.4 \text{ J}$$

Example (4-13)

A handful of copper shot is heated to 90 °C and then dropped into 80g of water at 10°C. The final temperature of the mixture is 18 °C. What was the mass of the shot?

Answer

Copper shot temperature (T_s) = 90 °C, water temperature (T_w) = 10 °C,
 Equilibrium temperature (T_e) = 18 °C, $m_w = 80$ g, $m_s = ?$
 Heat lost by shot = heat gained by water

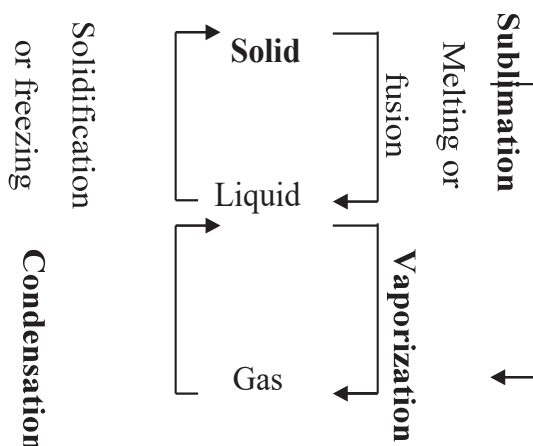
$$m_s c_s \Delta T_s = m_w c_w \Delta T_w$$

$$m_s (0.093) (90 - 18) = (80) (1) (18 - 10)$$

$$\therefore m_s = 95.6 \text{ g}$$

4.6. Change of Phase

A substance often undergoes a change in temperature when energy transferred between it and its surroundings. There are situations, however, in which the transfer of energy does not result in a change in temperature. This is the case whenever the physical characteristics of the substance change from one form to another; such a change is commonly referred to as a **phase change**. Two common phase changes are from to solid (melting) and from liquid to gas (boiling); another is a change in the crystalline structure of a solid. All such phase changes involve a change in internal energy but no change in temperature. The increase in internal energy in boiling, for example, is represented by breaking of bonds between molecules in the liquid state; this bond breaking allows the molecules to move farther apart in the gaseous state, with a corresponding increase in intermolecular potential energy.



As you might expect, different substances respond differently to the addition or removal of energy as they change phase because their internal molecular arrangements vary. Also, the amount of energy transferred during a phase change depends on the amount of substance involved. (It takes less energy to melt an ice cube than it does to thaw a frozen lake). If a quantity Q of energy transfer is required to change the phase of a mass m of a substance, the ratio $L = Q/m$ characterizes an important thermal properties of a substance. Because this added or removed energy does not result in temperature change, the quantity L is called the **latent heat** (literally, the "hidden" heat) of the substance. The value of L for a substance depends on the nature of the phase change, as well as on the properties of the substance.

Latent heat defined as the amount of heat per unit mass needed to transform material completely from one phase to another phase without change in temperature (i.e. at constant temperature).

4.6.1. Latent Heat of Fusion L_f

Defined as the amount of heat per unit mass needed to transform material completely from solid phase to liquid phase without change in temperature (i.e. at constant temperature called the melting temperature).

$$L_f = \frac{Q}{m} \quad \text{or} \quad Q = m L_f$$

4.6.2. Latent Heat of Vaporization L_v

Defined as the amount of heat per unit mass needed to transform material completely from liquid phase to gas phase without change in temperature (i.e. at const temperature called the boiling temperature).

$$L_v = \frac{Q}{m} \quad \text{or} \quad Q = m L_v$$

To understand the role of latent heat in phase change, consider the energy required to convert a 1.00 gram block of ice at -30°C to steam at 130°C . As shown in figure (4.14) the experimental results obtained when energy gradually added to ice. Let us examine each portion of the curves indicated by letters (A, B, C, D and E)

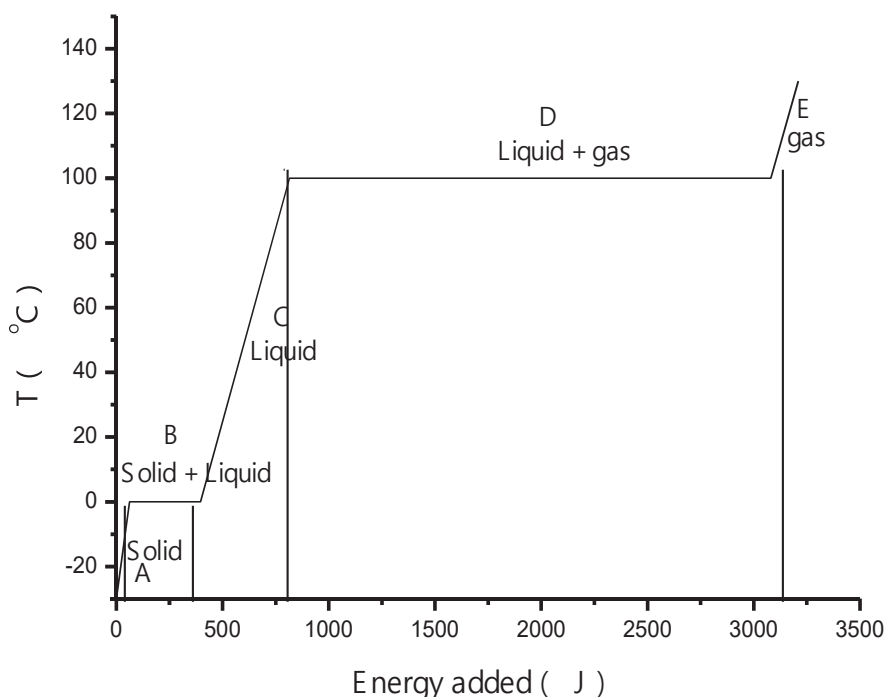


Figure (4.14) A plot of temperature versus energy added for a cube of ice at -30°C

Part A:

On this portion the temperature of ice (Solid phase) raised from -30°C to 0°C without change in state i.e. ice remains solid and because the specific heat of ice $2090\text{ J/kg}\cdot^{\circ}\text{C}$ we can calculate the heat added from the relation

$$Q = mc\Delta T = 1 \times 10^{-3} \times 2090 \times (0 - (-30)) = 62.7\text{ J}$$

Part B:

When the temperature of ice reach 0°C , the ice start to melt and a mixture of ice-water is present at constant temperature called melting point until added energy can melt all ice.

A quantity of heat $Q = m L_f = 1 \times 10^{-3} \times 3.33 \times 10^5 = 333\text{ J}$.

Part C:

On this portion the temperature of melted ice (water in liquid phase) raised from 0°C to 100°C without change in state, specific heat of water $4190\text{ J/kg}\cdot^{\circ}\text{C}$. The quantity of energy added

$$Q = mc\Delta T = 1 \times 10^{-3} \times 4190 \times (100 - 0) = 419.0\text{ J}$$

Part D:

When the temperature of water reaches 100 °C, the water starts to boil and a mixture of water-vapor is present at constant temperature called boiling point until added energy can vaporize all water.

A quantity of heat $Q = m L_v = 1 \times 10^{-3} \times 2.26 \times 10^6 = 2260 J$

Part E:

On this portion the temperature of vapor (gas phase) raised from 100°C to 120°C without change in state, specific heat of water 2010 J/kg.°C. The quantity of energy added

$$Q = m c \Delta T = 1 \times 10^{-3} \times 2010 \times (120 - 100) = 40.2 J$$

The total amount of heat needed for this process can be calculated from all this process such as

$$Q_t = Q_1 + Q_2 + Q_3 + Q_4 + Q_5$$

$$Q_t = 62.7 + 333.0 + 419.0 + 2260.0 + 40.2 = 3110.9 J$$

We can describe See figure (4.15) phase change

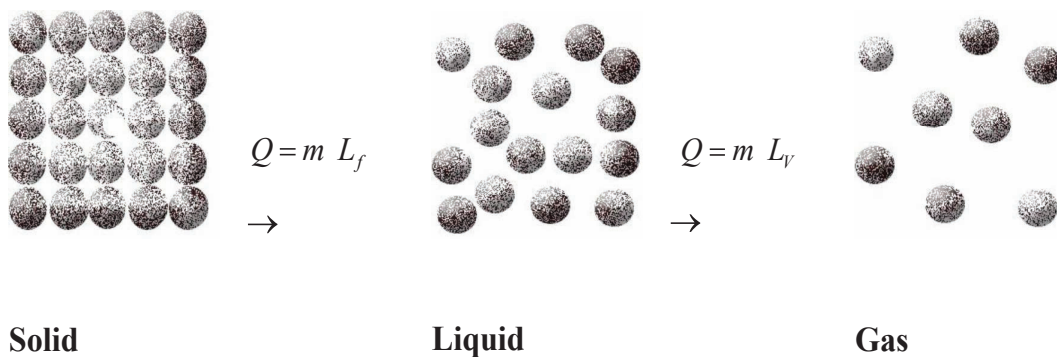


Figure (4.15) Simple model illustrates the change of phases.

Example 4.14

What mass of steam initially at 130 °C needed to warm 200g of water in a 100 g glass container from 20°C to 50°C. (Specific heat of steam = 2010 J/kg.K, Specific heat of water = 4190 J/kg.K, $L_v = 2.26 \times 10^6$ J/kg, specific heat of glass = 837 J/kg.K)

Solution

Steam losses energy in three stages;

$$1) \text{ Steam cools to } 100 \quad Q_1 = m_s c_s \Delta T_s = m_s \times 2010 \times (130 - 100) = 6.03 \times 10^4 m$$

Steam at 100 condensed and converted into water at 100 °C

$$Q_2 = m_s L_v = m_s \times 2,26 \times 10^6$$

- 1) Temperature of water created from steam cools to 50 °C

$$Q_3 = m_w c_w \Delta T_w = m_s \times 4190 \times (100 - 50) = 2.09 \times 10^5 m_s$$

- 2) Adding these three stages gives us the heat lost from steam

$$Q_T = Q_1 + Q_2 + Q_3 = 6.03 \times 10^4 m_s + m_s \times 2,26 \times 10^6 + 2.09 \times 10^5 m_s = 2.53 \times 10^6 m_s$$

- 3) The temperature increases in both Liquid and glasses, so the heat gained $Q = Q_w + Q_g$

$$Q = m_w c_w \Delta T_w + m_g c_g \Delta T_g = 0.200 \times 4190 \times (50 - 20) + 0.100 \times 837 \times (50 - 20) = 2.77 \times 10^4 J$$

Heat gained = heat lost

$$2.53 \times 10^6 m_s = 2.77 \times 10^4 J \Rightarrow m_s = 1.09 \times 10^{-2} Kg$$

Example 4.15

Liquid helium has a very low boiling point 4.2 K, and very low latent heat of vaporization. 2.09×10^4 J/kg. If energy is transferred to a container of boiling liquid helium from an immersed electric heater at a rate of 10 W (J/s), how long does it take to boil away 1.0 kg of the liquid. ($L_v = 2.09 \times 10^4$ J/kg).

Solution

The quantity of heat needed to boil unit mass of He

$$Q_2 = m_s L_v = 1 \times 2.09 \times 10^4 = 2.09 \times 10^4 J$$

The rate of energy supply

$$\frac{Q}{t} = \frac{2.09 \times 10^4}{t} \Rightarrow 10 \frac{J}{s} = \frac{2.09 \times 10^4}{t} \Rightarrow t = 2.09 \times 10^3 \text{ sec} = 34.83 \text{ min}$$

4.7. Heat Transfer

Heat may be transferred from one point to another by conduction, convection or radiation see figure (4.16)

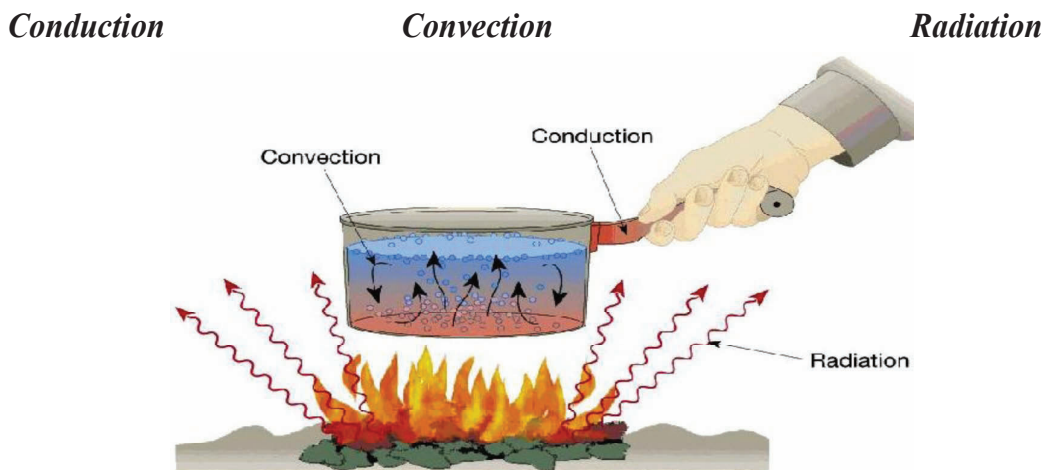


Figure (4.16)

4.7.1. Conduction

It requires physical contact between the bodies and it also requires a physical medium.

* A good conductor of electricity is also a good conductor of heat.

The **Wiedemann-Franz law** expresses the observation that the ratio k/σ is nearly the same for all metals. This is a reflection of thermal conduction in metals being largely due to the movement of the same free electrons as those which are responsible for electrical conduction in metals.

4.7.1.1. Rate of Heat Transfer (R)

It is the quantity of heat (Q) that flows perpendicular to the face during a time (t)

$$R = \frac{Q}{t} \quad \text{or} \quad R = \frac{dQ}{dt} \quad \text{J/s} \quad \text{or} \quad \text{W}$$

*The quantity of heat transferred by unit time (R) is:

- Directly proportional to the area A.
- Directly proportional to temperature difference $\Delta T = T_h - T_c$

Inversely proportional to thickness Δx , See figure (4.17)

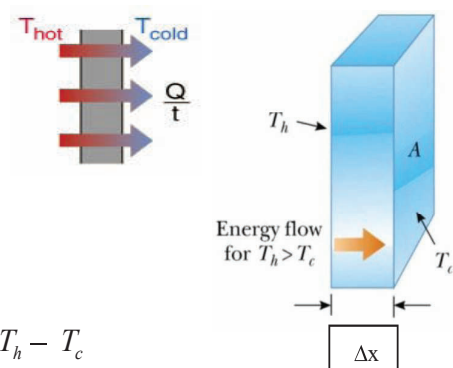


figure (4.17)

$$R \propto \frac{A \Delta T}{\Delta x} \Rightarrow R = k A \frac{\Delta T}{\Delta x} \Rightarrow R = k A \frac{T_h - T_c}{\Delta x}$$

Where:

- The thermal conductivity (k)

It is the measure of its ability of the substance to conduct heat.

OR, The rate at which the heat flows through a certain area of a body.

- $|dT / dx|$ is called the temperature gradient of the material
- For a rod of length L, the temperature gradient can be expressed as:

$$\left| \frac{dT}{dx} \right| = \frac{T_h - T_c}{L}$$

❖ Rate of Heat Transfer (R) for rod of length L is:

$$R = k A \frac{T_h - T_c}{L}$$

N.B

When two slabs of different thermal conductivity and similar cross sections are connected (in contact) see figure (4.18), the rate at which heat is conducted through each material must be constant.

Analogy between heat conduction and electrical systems

$$R = k A \frac{\Delta T}{L} \Rightarrow R = A \frac{T_h - T_c}{r} \Rightarrow r = \frac{L}{k}$$

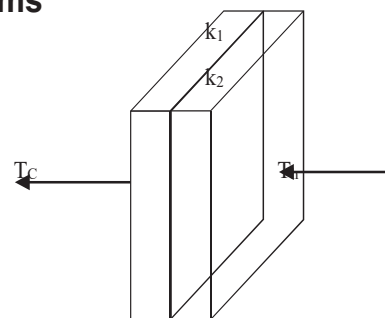


Figure (4.18)

4.7.1.1.1. For one slab, See figure (4.19)

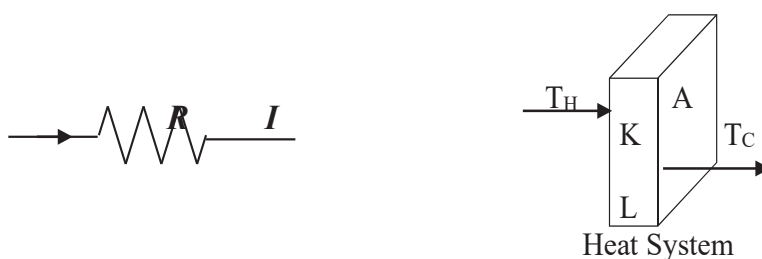


Figure (4.19)

$$R = k A \frac{\Delta T}{L} \Rightarrow R = A \frac{T_h - T_c}{r} \Rightarrow r = \frac{L}{k}$$

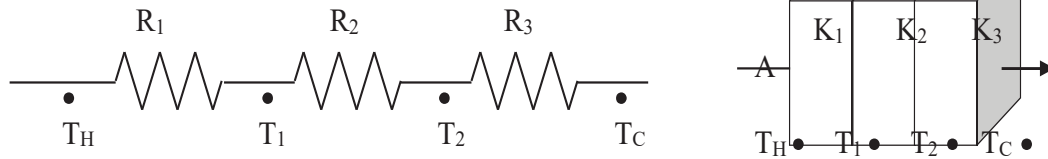
4.7.1.1.2. For several slabs(a) Series, See figure (4.20)

Figure (4.20)

$$H = A \frac{\Delta T_{\text{overall}}}{\Sigma R_{\text{th}}} = A \frac{T_H - T_C}{R_1 + R_2 + R_3}$$

$$\text{Where } R = \frac{L}{k}$$

$$\text{* Total resistance for series connection: } R_T = R_1 + R_2 + R_3 + \dots + R_n$$

Example (4.16)

A wall consisting of four layers, with thermal conductivity $k_1=0.06\text{W/m.K}$, $k_3=0.04\text{W/m.K}$, and $k_4=0.12\text{W/m.K}$. the layer thicknesses are $L_1=1.2\text{cm}$, $L_3=5.6\text{cm}$, and $L_4=4\text{cm}$. The known temperatures are $T_1=30^\circ\text{C}$, $T_{12}=25^\circ\text{C}$, and $T_4=-10^\circ\text{C}$. energy transfer through the wall is steady. What is interface temperature T_{34} ?

Solution

$$\frac{R}{A} = k_1 \frac{T_h - T_1}{L_1} = k_4 \frac{T - T_c}{L_4}$$

Where $T_h = 30^\circ\text{C}$, $T_1 = 25^\circ\text{C}$ and $T_c = -10^\circ\text{C}$. We solve for the unknown T .

$$T = T_c + \frac{k_1 L_4 T_h}{k_4 L_1} (T_h - T_1) = -10 + \frac{(0.06)(4 \times 10^{-2})}{(0.12)(1.2 \times 10^{-2})} (30 - 25) = -1.7^\circ\text{C}$$

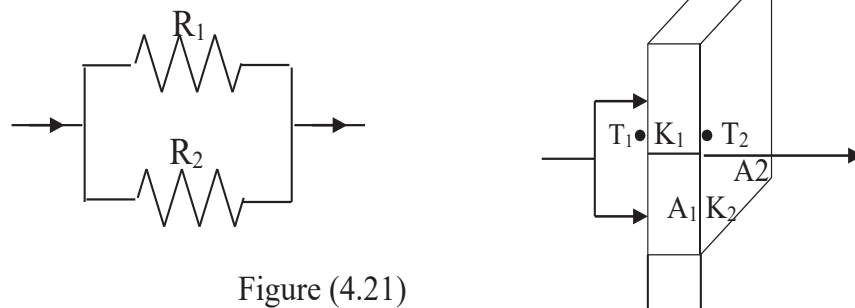
(b) Parallel, See figure (4.21)

Figure (4.21)

Example (4.17)

A Styrofoam box used to keep drinks cold at a picnic has total wall area (including the lid) of 0.8 m^2 and wall thickness of 2 cm . It is filled with ice, water, and cans of cola at 0°C . What is the rate of heat flow into the box if the temperature of the outside wall is 30°C ?

Answer

$$A = 0.80 \text{ m}^2, \quad L = 2 \text{ cm}, \quad T_c = 0^\circ\text{C}, \quad T_H = 30^\circ\text{C}, \quad k = 0.01 \text{ W/m} \cdot \text{K}$$

$$R = kA \frac{T_h - T_c}{L} = 0.01 \times 0.8 \frac{30 - 0}{2 \times 10^{-2}} = 12 \text{ W}$$

$$Q = H \times t = 12 \times 1 \times 24 \times 3600 = 1.04 \times 10^6 \text{ J}$$

4.7.2. Convection

It occurs when a liquid or gas is in contact with a solid body at a different temperature. I.e. it is required a moving medium see figure (4.22)

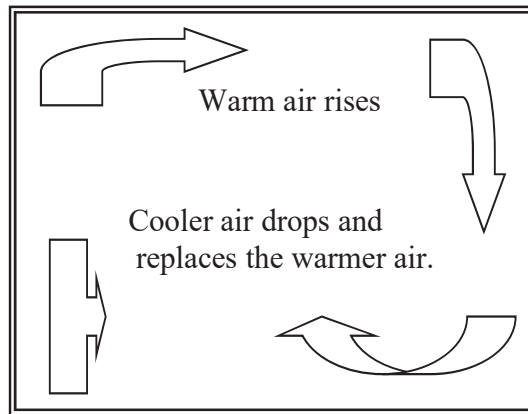


Figure (4.22)

Convection

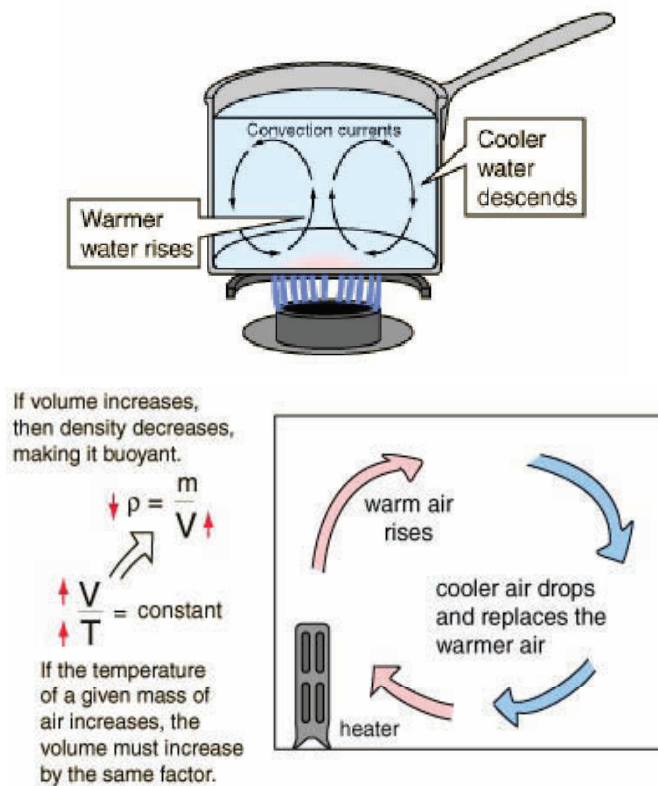


Figure (4.23)

- It is a heat transfer by mass motion of a fluid diffusion such as air or water when the heated fluid is caused to move away from the source of heat carrying energy with it.

-The particles of liquid near the bottom has higher temperature; and low density. So; this hot particles move upward toward the surface of the liquid. At the same time, the cold particles (with high density) move downward toward the bottom of liquid. see figure (4.24)

-The types of Convection:

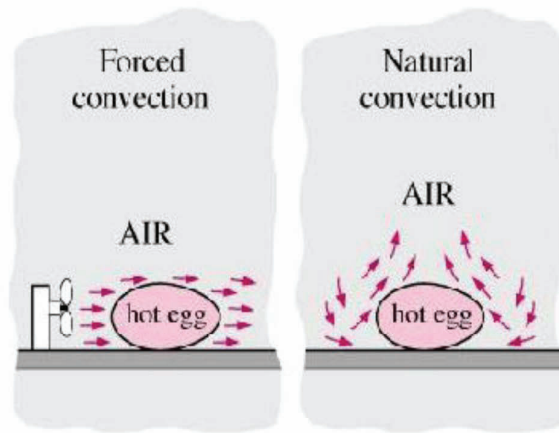


Figure (4.24)

4.7.2.1. Natural Convection:

in which the motion is caused by the difference in density.

In gravitational field, the hotter, lighter fluid rises while the colder, heavier fluid sinks, figure (4.24)

4.7.2.2. Forced Convection:

in which the motion is caused by the action of pump or fan.

*The rate of heat flow (or power) (P) is proportional to the area (A) and temperature difference ΔT . figure (4.20)

$$P = (Q / t) = h A \Delta T = h A (T_{\text{surface}} - T_{\text{fluid}})$$

Where h , is the convection coefficient.

- Convection is also the process of conduction between a solid surface and moving liquid.

4.7.3. Radiation

It does not require physical contact between the bodies and so the heat energy is transferred by electromagnetic waves. See figure (4.25)

Radiation

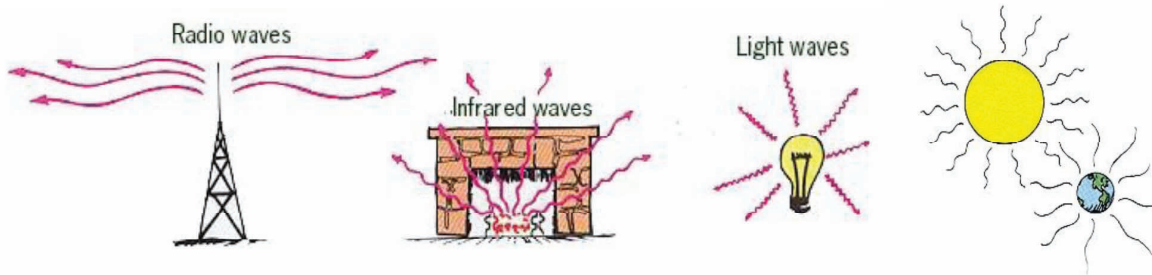


figure (4.25)

- Thermal radiation

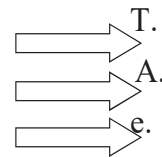
It is the transfer of heat energy by electromagnetic radiation, which carry energy away from the emitting object.

4.7.3.1.Blackbody Radiation (or Cavity Radiation)

It is the object (or system) which absorbs all radiation (energy) incident on it and reradiate energy which is characteristic of the radiating system.

*** The factors affecting on the rate of heat transfer are:

- (1) The absolute temperature of the radiating body.
- (2) The nature of the exposed surfaces.
- (3) The emissivity of the radiating body.



***Emissivity (ε):** is a measure of body's ability to absorb or emit thermal radiation. (it has a numerical value between 0.0 and 1.0).

***The rate of radiation of a body (P/A):** is the radiant energy emitted per unit area per unit time.

4.7.3.2.Stefan's law:

“The rate at which an object radiates energy is proportional to the fourth power of its temperature”

* The rate of radiated energy or radiated power (P) = $\epsilon A \sigma T^4$

Where, Stefan's Constant (σ) = $5.67 \times 10^{-8} \text{ W / m}^2 \cdot \text{K}^4$

A : is the surface area , ϵ : is the emissivity

T : is the absolute temperature (in Kelvin)

4.7.3.3. Net Energy Gained or Lost by An Object

A body at the absolute temperature (T) is radiating, also, the surroundings of the body at temperature (T_s) radiate and the body absorbs some of this radiation.

- P_{net} : Net energy gained or lost per second by the object as a result of radiation, figure (4.26)

$$P_{\text{net}} = \varepsilon A \sigma T^4 - \varepsilon A \sigma T_s^4 = \varepsilon A \sigma (T^4 - T_s^4)$$

Where, Stefan's Constant (s) = $5.67 \times 10^{-8} \text{ W} / \text{m}^2 \cdot \text{K}^4$

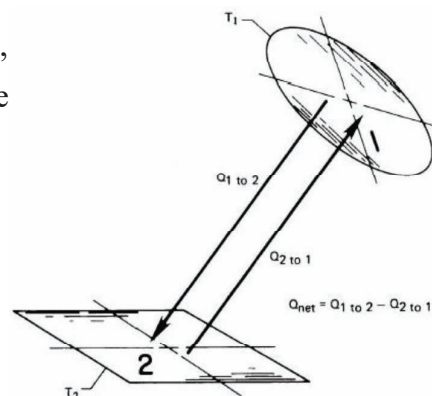


figure (4.26)

A : is the surface area , ε : is the emissivity

T : is the absolute temperature (in Kelvin)

Example (4.18)

A thin square steel plate, 10 cm on a side, is heated in a blacksmith's forge to a temperature 800°C . If the emissivity is 0.6, what is the total rate of radiation of the energy ?

Where, Stefan's Constant (s) = $5.67 \times 10^{-8} \text{ W} / \text{m}^2 \cdot \text{K}^4$

Answer

A thin square steel plate 10 cm on a side, $T = 800 + 273 = 1073 \text{ K}$, $\varepsilon = 0.60$

$$P = A \varepsilon \sigma T^4 = \left[2 \times (10 \times 10^{-2})^2 \right] \times 0.6 \times 5.67 \times 10^{-8} \times (1073)^4 = 902 \text{ W}$$

4.8. The First Law of Thermodynamics

4.8.1. Work in Thermodynamics

- ❖ Thermodynamics is concerned with the work done by (or on) the system as well as the heat energy exchanged between the system and the surrounding medium
- ❖ We will always consider that most of the operations that will be performed are quasi-static processes, and therefore the resulting changes in volume, pressure, temperature, number of moles and ... are slight and slow changes, See figure (4.27)

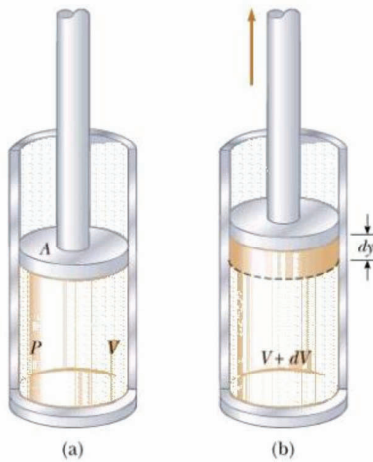


Figure (4.27)

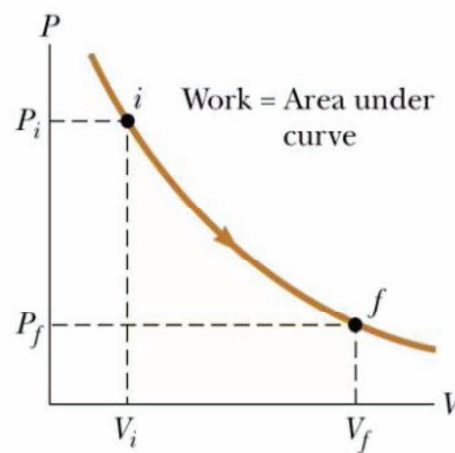


Figure (4.28)

- So,

$$dW = PdV$$

$$W = \int_{V_1}^{V_2} P dV$$

- The work is the area under the P-V curve, figure (4.28)
- The work done by (or on) the system is not only depend on the initial and final states but also depend on the path of the process.

For example, we assume that the system moves from the initial state to the final state through the three paths shown in the figure (4.29)

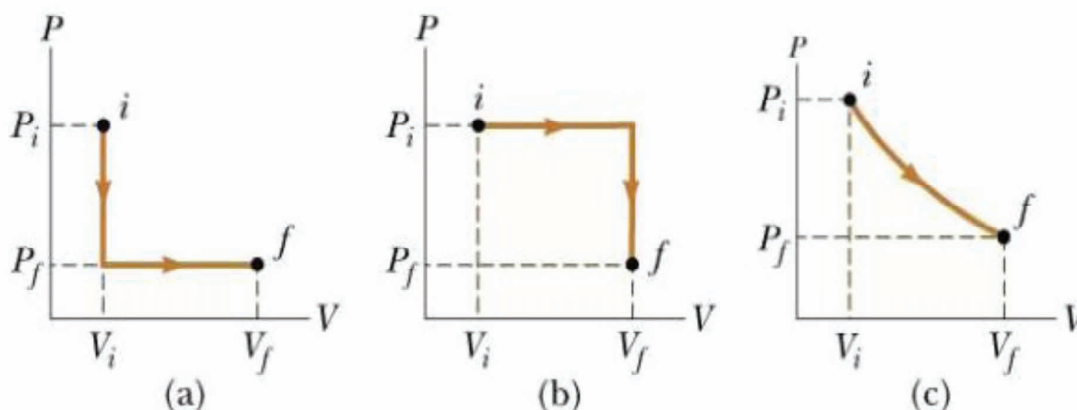


Figure (4.29)

- The work; for the case (a) $W = P_f(V_f - V_i)$
- The work; for the case (b) $W = P_i(V_f - V_i)$
- The work; for the case (c) $dW = PdV \Rightarrow W = \int_{V_i}^{V_f} P dV$

Note that: the work In the previous three cases the value is not equal, so its value depends on the path.

4.8.2. The First Law of Thermodynamics (Energy Conservation)

“The internal energy of a system changes when work is done on the system (or by it), and when it exchanges heat with the environment”

*Mathematical form, $\Delta U = Q - W \quad \Longleftrightarrow \quad \text{OR} \quad \Longleftrightarrow \quad dU = dQ - dW$

Where, ΔU : is the change in internal energy

Note that: The change in internal energy is a function that does not depend on the path, but depends only on the initial and final states

W : is the work done(by or on) the system.

Q : is the exchange heat (added or removed) between the system and the surroundings.

Q $\longrightarrow$ $\left\{ \begin{array}{l} (+) \text{ Heat added, supplied or absorbed.} \\ (-) \text{ Heat removed or lost.} \end{array} \right.$

W $\longrightarrow$ $\left\{ \begin{array}{l} (+) \text{ Work done by the system or the system expanded.} \\ (-) \text{ Work done on the system or the system compressed.} \end{array} \right.$

Thermodynamic Process

A thermodynamic process is when heat moves, either within systems or between systems. There are four types of idealized thermodynamic processes. They are

4.8.2.1. An Isobaric Process

It is a process during which the system's pressure remains constant. See figure (4.30)

Find the work done by the gas:

$$W = \int_{V_i}^{V_f} P \, dV = P \int_{V_i}^{V_f} dV = P (V_f - V_i)$$

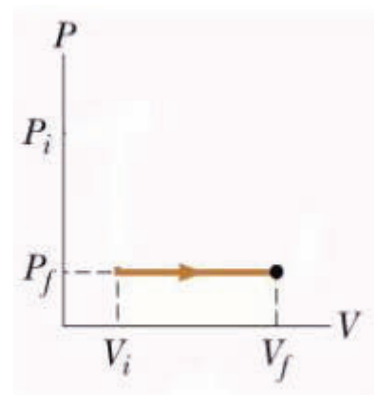


Figure (4.30)

4.8.2.2. An Isothermal Process for Ideal Gas)

It is a process during which the system's temperature remains constant. See figure (4.31)

$$\Delta U = 0$$

$$Q = W$$

- Find the work done by the gas to increase its volume from V_i to V_f at constant temperature?

Since, $P V = n R T$

So, $P = \frac{n R T}{V}$

$$W = \int_{V_i}^{V_f} P \, dV = \int_{V_i}^{V_f} \frac{n R T}{V} \, dV = n R T \ln \frac{V_f}{V_i}.$$

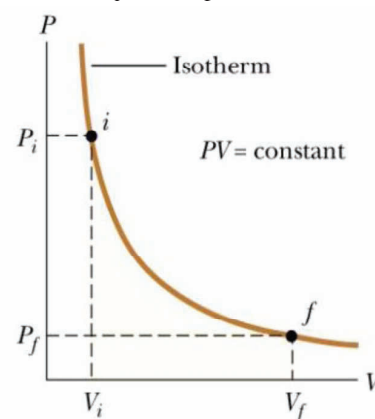


figure (4.31)

4.8.2.3. An Isochoric Process

It is a process during which the system's volume remains constant.

See figure (4.32)

$$\Delta V = 0$$

$$W = 0$$

The change in internal energy;

$$\Delta U = Q$$

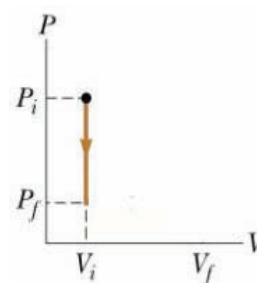


Figure (4.32)

4.8.2.4. For Adiabatic Process

No heat exchanged between the system and the surroundings, see figure (4.33)

$$Q = 0$$

$$\Delta U = W$$

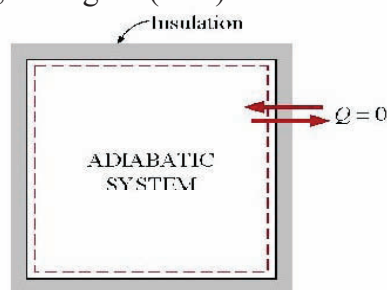


figure (4.33)

4.8.2.5. For Cyclical Process

No change in system's temperature.

There is no change in the temperature of the system (because the system starts from a state and after a set of consecutive procedures ends in the same case) and thus the internal energy is constant and does not change.

$$\Delta U = 0$$

$$Q = W$$

4.8.2.6. For Isolated Process

No heat exchanged and there is no work done on the external environment

$$Q = 0$$

$$W = 0$$

$$\Delta U = 0$$

Example (4.19)

Three moles of helium are initially at 20 ° C, and a pressure of 1 atm. What is the work done by the gas if the volume is doubled : (a) at constant pressure, or (b) isothermally?

If the $R = 8.31 \text{ J/mol.k}$, $P = 1 \text{ atm} = 101.3 \times 10^3 \text{ pascal}$

Solution

a) Since, $P V = n R T$

So, $V_1 = [3 \times 8.31 \times 293] / [101.3 \times 10^3] = 0.072 \text{ m}^3$

The work $W = P(V_2 - V_1) = 101.3 \times 10^3 (2 \times 0.072 - 0.072) = 7294 \text{ J}$

$$\text{b) } W = \int_{V_i}^{V_f} P dV = \int_{V_i}^{V_f} \frac{n R T}{V} dV = n R T \ln \frac{V_f}{V_i}.$$

$$W = n R T \ln \frac{V_f}{V_i} = 3 \times 8.31 \times 293 \times \ln \frac{2 V_i}{V_i} = 5063 \text{ J}$$

Example (4.20)

One mole of oxygen expand at constant temperature $T = 310 \text{ K}$ from an initial volume of 12 Lit to final volume of 19 Lit.

a) How much work is done by the gas?

b) How much work is done by the gas during an isothermal compression from volume of 19 Lit to volume of 12 Lit?

Solution

$$W = n R T \ln \frac{V_f}{V_i} = 1 \times 8.31 \times 310 \times \ln \frac{19}{12} = 1184 \text{ J}$$

$$W = n R T \ln \frac{V_f}{V_i} = 1 \times 8.31 \times 310 \times \ln \frac{12}{19} = -1184 \text{ J}$$

Example (4.21)

Calculate the work done by 1 mole of an ideal gas that is kept at 0°C in an expansion from 3 Lit to 10 Lit ?

Solution

$$W = n R T \ln \frac{V_f}{V_i} = 1 \times 8.31 \times 273 \times \ln \frac{10}{3} = 2731 \text{ J}$$

4.9. Problems

Choose The Correct Answer in Each of The Followings:

- 1) The property that determines whether an object is in thermal equilibrium with other objects.
 - A) Thermal expansion
 - B) Pressure
 - C) Temperature
 - D) Thermal equilibrium

- 2) Kelvin temperature scale Thermometer calibrated due to
 - A) Triple point of water
 - B) Absolute zero
 - C) Ice point
 - D) (A and B)

- 3) the increase of steel sphere volume is $1.67 \times 10^{-2} \text{ m}^3$ due to the increase of temperature from -15°C to 55°C , if its radius is 1 m then the average coefficient of volume expansion equals
 - A) 5×10^{-3}
 - B) 11×10^{-6}
 - C) 33×10^{-6}
 - D) 22×10^{-3}

- 4) It does not require physical contact between the bodies and the heat energy is transferred by electromagnetic waves
 - A) Conduction
 - B) Radiation
 - C) Convection
 - D) all

- 5) Celsius temperature scale Thermometer calibrated due to
 - A) ice melting (ice point)
 - B) water boiling (steam point)
 - C) mercury boiling point
 - D) (A and B)

- 6) The surface expansion of a square ($l = 3 \text{ m}$) of lead ($\alpha = 29 \times 10^{-6} \text{ }^\circ\text{C}^{-1}$) is 0.0261 m^2 , then the increase of temperature
 - A) 25°C
 - B) 40°C
 - C) 50°C
 - D) 65°C

7) Suppose object C is in thermal equilibrium with object A and with object B. The zeroth law of thermodynamics states:

- A) that C will always be in thermal equilibrium with both A and B
- B) that C must transfer energy to both A and B
- C) that A is in thermal equilibrium with B
- D) that A cannot be in thermal equilibrium with B

8) The coefficient of linear expansion of steel is 11×10^{-6} per $^{\circ}\text{C}$. A steel ball has a volume of exactly 200 cm^3 at 0°C . When heated to 100°C its volume becomes(in cm^3) :

- A) 200.66
- B) 200.11
- C) 200.0011
- D) 200.0033

9) The temperature difference between the inside and the outside of a home on a cold winter day is 57°F , the difference on the Celsius and the Kelvin scales respectively are.

- A) 31.67 , 31.67
- B) 13.89 , 286.89
- C) 57 , 57
- D) 31.67 , 304.67

10) An annular ring of aluminum is cut from an aluminum sheet as shown. When this ring is heated:



- A) The aluminum expands outward and the hole remains the same in size
- B) The hole decreases in diameter
- C) The area of the hole expands the same percent as any area of the aluminum
- D) The area of the hole expands a greater percent than any area of the aluminum

11). A calorie is about:

- A) 0.24 J
- B) 8.3 J
- C) 250 J
- D) 4.2 J

12) The amount of energy per unit mass that must be transferred as heat when a sample completely undergoes a phase change is called the

- A) The specific heat
- B) The heat capacity
- C) Latent heat

D) The temperature

13) The concrete sections ($\alpha = 12 \times 10^{-6} \text{ }^\circ\text{C}^{-1}$) are designed to have a length of 10.0 m. The sections are poured and cured at $20 \text{ }^\circ\text{C}$. What minimum spacing (in m) should the engineer leave between the sections to eliminate buckling if the concrete is to reach a temperature of $50 \text{ }^\circ\text{C}$?

A) $3.6 \times 10^{-3} \text{ m}$

B) $6.3 \times 10^{-3} \text{ m}$

C) $3 \times 10^{-3} \text{ m}$

D) $8.4 \times 10^{-3} \text{ m}$

14) A cube of aluminum has an edge length of 20 cm. Aluminum has a density 2.7 g/cm^3 and a specific heat $0.217 \text{ cal/g } ^\circ\text{C}$. When the internal energy of the cube increases by 47000 cal its temperature increases by:

A) $5 \text{ }^\circ\text{C}$

B) $10 \text{ }^\circ\text{C}$

C) $20 \text{ }^\circ\text{C}$

D) $100 \text{ }^\circ\text{C}$

15) Possible units for the coefficient of volume expansion are:

A) $\text{mm}/^\circ\text{C}$

B) $\text{mm}^3/^\circ\text{C}$

C) $(^\circ\text{C})^3$

D) $1/^\circ\text{C}$

16) When the temperature of a copper penny is increased by $100 \text{ }^\circ\text{C}$, its diameter increases by 0.17%. The area of one of its faces increases by:

A) 0.17%

B) 0.34%

C) 0.51%

D) 0.13%

17) How many calories are required to change one gram of $0 \text{ }^\circ\text{C}$ ice to $100 \text{ }^\circ\text{C}$ steam? The latent heat of fusion is 80 cal/g and the latent heat of vaporization is 540 cal/g . The specific heat of water is $1 \text{ cal/g } ^\circ\text{C}$

A) 100 cal

B) 540 cal

C) 620 cal

D) 720 cal

18) The specific heat of lead is $0.03 \text{ cal/g } ^\circ\text{C}$. 300 g of lead shot at $100 \text{ }^\circ\text{C}$ is mixed with 100 g of water at $70 \text{ }^\circ\text{C}$ ($c_w = 1 \text{ cal/g } ^\circ\text{C}$) in an insulated container.

The final temperature of the mixture is:

A) $100 \text{ }^\circ\text{C}$

B) $85.5 \text{ }^\circ\text{C}$

C) $79.5 \text{ }^\circ\text{C}$

D) $72.5 \text{ }^\circ\text{C}$

- 19) A Celsius thermometer and a Fahrenheit thermometer both give the same reading for a certain sample. The corresponding Kelvin temperature is:
- A) - 40 K
 - B) 40 K
 - C) 301K
 - D) 233.15 K
- 20) A Kelvin thermometer and a Fahrenheit thermometer both give the same reading for a certain sample. The corresponding Celsius temperature is:
- A) 574 °C
 - B) 232 °C
 - C) 301 °C
 - D) 614 °C
- 21) The specific heat of a substance is:
- A) The amount of heat energy to change the state of one gram of the substance
 - B) The amount of heat energy per unit mass emitted by oxidizing the substance
 - C) The amount of heat energy per unit mass to raise the substance from its freezing to its boiling point
 - D) The amount of heat energy per unit mass to raise the temperature of the substance by 1°C
- 22) The coefficient of linear expansion of iron is 10^{-5} per °C. The volume of an iron cube, 5 cm on edge, will increase by what amount(in cm^3) if it is heated from 10 °C to 60 °C?
- A) 0.00375
 - B) 0.1875
 - C) 0.0225
 - D) 0.0075
- 23) The zeroth law of thermodynamics allows us to define:
- A) Work
 - B) Pressure
 - C) Temperature
 - D) Thermal equilibrium
- 24) Which one of the following statements is true?
- A) Temperatures differing by 25° on the Fahrenheit scale must differ by 45° on the Celsius scale
 - B) 40K corresponds to -40 °C
 - C) Temperatures which differ by 10°C on the Celsius scale must differ by 18°F on the Fahrenheit scale
 - D) Water at 90 °C is warmer than water at 202 °F
- 25) If two objects are in thermal equilibrium with each other:
- A) They cannot be moving
 - B) They cannot be undergoing an elastic collision
 - C) They cannot have different pressures
 - D) They cannot be at different temperatures

26) The coefficient of linear expansion of iron is 1.0×10^{-5} per $^{\circ}\text{C}$. The surface area of an iron cube, with an edge length of 5.0 cm, will increase by what amount (in cm^2) if it is heated from 10°C to 60°C ?

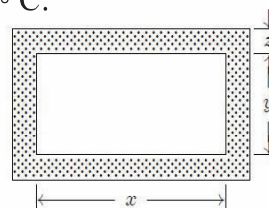
- A) 0.0125
- B) 0.025
- C) 0.075
- D) 0.15

27) A balloon is filled with cold air and placed in a warm room. It is NOT in thermal equilibrium with the air of the room until:

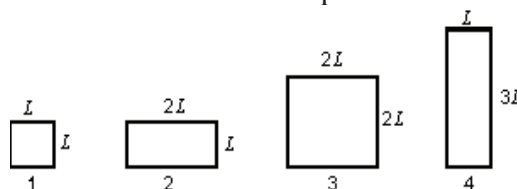
- A) it rises to the ceiling
- B) it sinks to the floor
- C) it stops expanding
- D) it starts to contract

28) The figure shows a rectangular brass plate at 0°C in which there is cut a rectangular hole of dimensions indicated. If the temperature of the plate is raised to 150°C :

- A) x will increase and y will decrease
- B) both x and y will decrease
- C) x will decrease and y will increase
- D) both x and y will increase



29) The diagram shows four rectangular plates and their dimensions. All are made of the same material. The temperature now increases. Of these plates:



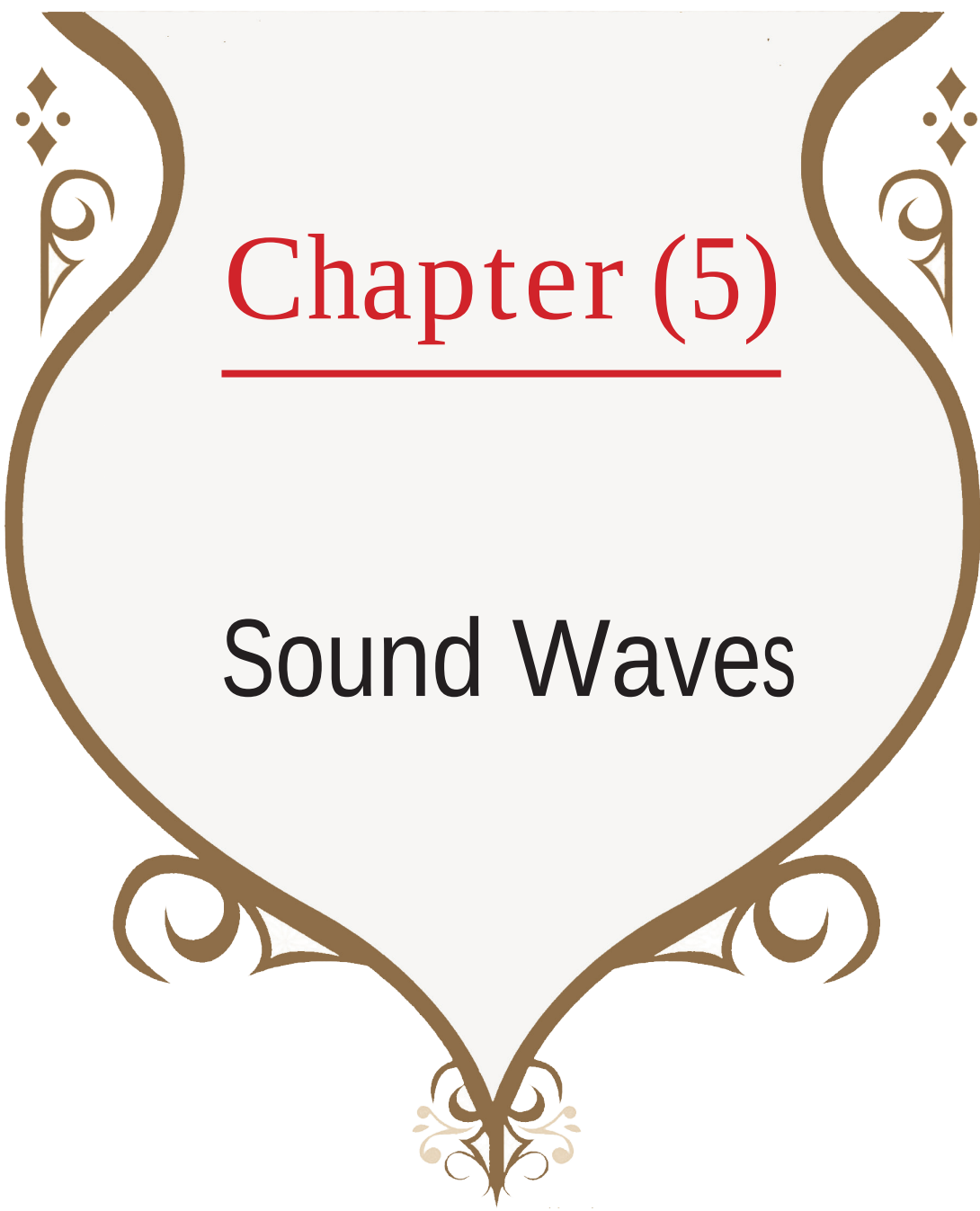
- A) The vertical dimension of plate 1 increases the most and the area of plate 1 increases the most
- B) The vertical dimension of plate 2 increases the most and the area of plate 4 increases the most
- C) The vertical dimension of plate 3 increases the most and the area of plate 1 increases the most
- D) The vertical dimension of plate 4 increases the most and the area of plate 3 increases the most

Q2:

1. What is 0°K on Celsius and Fahrenheit scale?
2. What is the room temperature of 72°F in Celsius scale?
3. What is the temperature change of 20°C in both $^{\circ}\text{F}$ and $^{\circ}\text{K}$ scale?

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Chapter (5)

Sound Waves



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Chapter (5)

Sound Waves

Content

What is a wave - Classifying Waves -Wave Properties -Types of sound waves – Speed of sound waves - The relation between frequency and wavelength – Intensity – The Doppler effect.

Learning Objective

1. Define the major properties of waves: frequency, period, wavelength, energy, amplitude and velocity, being able to use all of the units and symbols for each.
2. know and recall that the audio frequency range is approximately 20 Hz to 20 kHz;
3. Known the Intensity of sound waves – the Doppler effect

5.1. What Is a Wave?

- A wave is a disturbance that travels through a medium from one location to another.
- A wave carries energy through matter or space without transferring matter.
- There are two main types of waves:

5.1.1. Mechanical Waves: Examples (Water waves , Sound waves , Seismic waves)

- require a material medium to exist. (must have a substance to travel through)
- cannot travel in a vacuum

5.1.2. Electromagnetic Waves: Examples (Radio waves- Microwaves - Infrared, Visible and Ultraviolet light - X-rays and Gamma rays.

- do not require a material medium to travel through
- can travel in a vacuum

5.1.3. Classifying Waves:

- Transverse Waves (light waves)
- Longitudinal Waves (Sound waves)
- Transverse Waves involve oscillations perpendicular to the direction in which the waves travel. See figure (5.1)
- Longitudinal Waves involve oscillations parallel to the direction in which the waves travel. See figure (5.1)

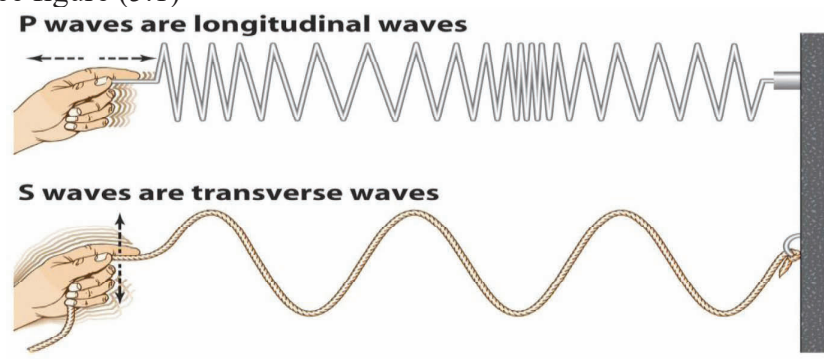


Figure (5.1)

Remark 1:**Parts of Transverse waves:**

- Crest: the highest point of the wave
- Trough: the lowest point of the wave See figure (5.2)

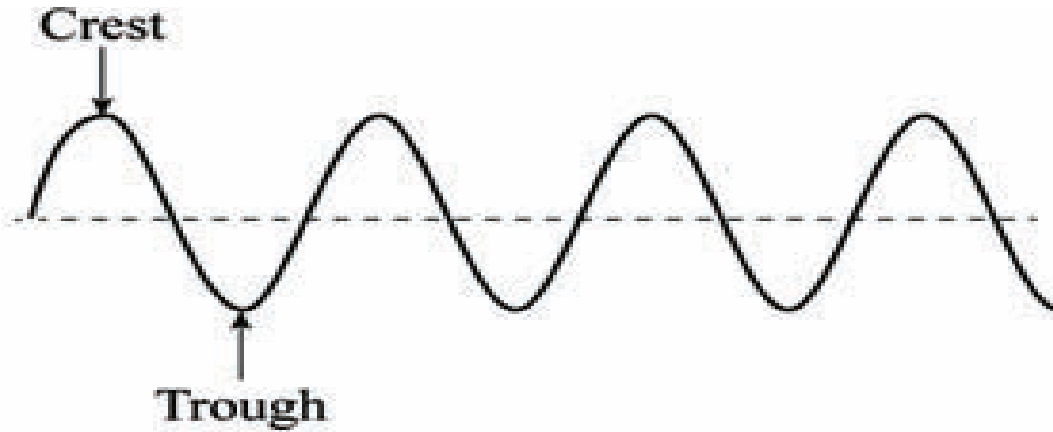


Figure (5.2)

Remark 2:**Parts of Longitudinal waves:**

- Compression: where the particles are close together
- Expansion: where the particles are spread apart See figure (5.3)

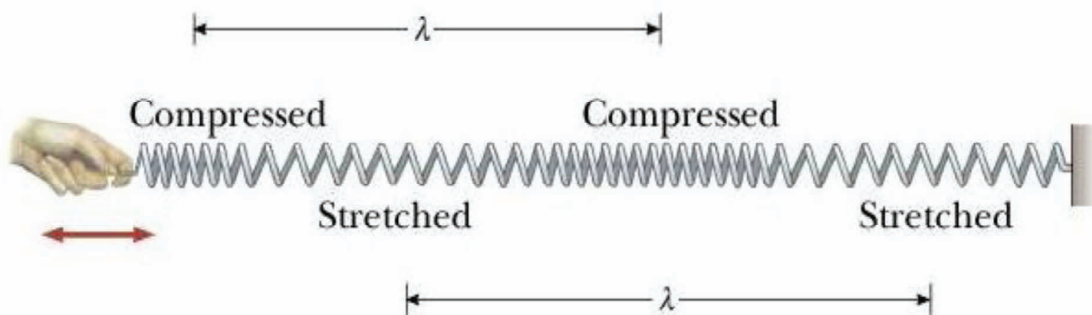


Figure (5.3) The sound wave consists of condensations and rarefactions.

Remark 3:**Traveling Sound Waves: See figure (5.4)**

- *Amplitude (A)* : Maximum displacement of particle of the medium from its equilibrium point.
- *Wavelength (λ)*: Distance from crest (max positive displacement) to crest; same as distance from trough (max negative displacement) to trough.
Wavenumber $k = 2\pi / \lambda$.
- *Frequency (f)* : The number of cycles passing by in a given time. The SI unit for frequency is the Hertz (Hz), which is one cycle per second.
Angular frequency $\omega = 2\pi f$

- *Period (T)* : the time required for one full cycle of the wave to pass by. Period is the reciprocal of frequency: $T = \frac{1}{f}$.

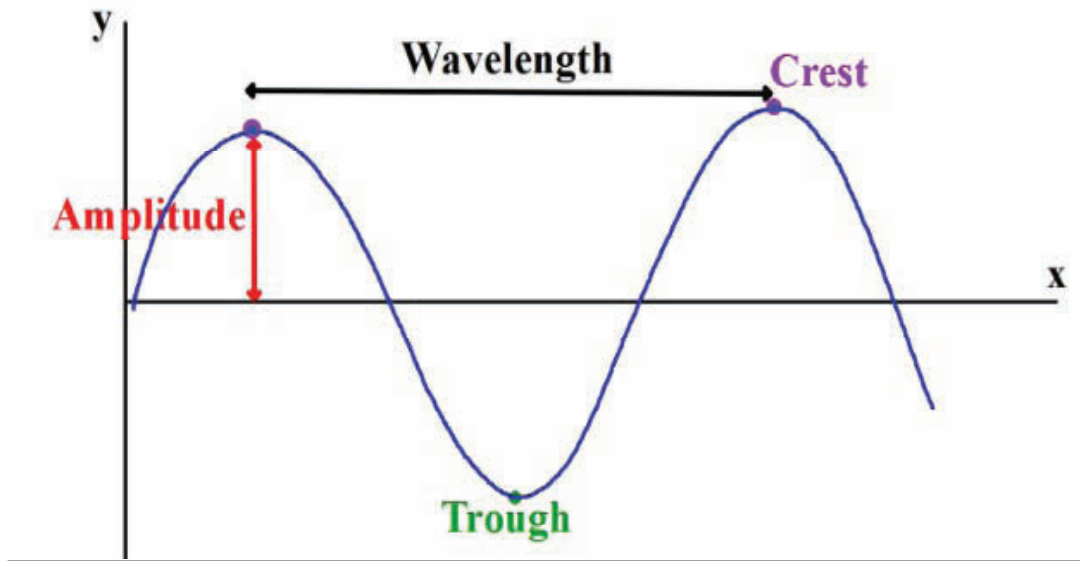


Figure (5.4)

5.2.Sound Waves

* The sound waves are the most important example of longitudinal waves, that travel through any medium with speed depends on the properties of the medium. See figure (5-3)

* The sound waves cannot be traveled through vacuum, because there is no material to transmit the condensations and rarefactions.

5.2.1.The Properties of Sound Waves:

- The sound waves are longitudinal waves.
- The sound waves are required a medium to transmit from position to another.
- The speed of sound waves in air at $0^\circ\text{C} = 331 \text{ m/sec}$.
- The speed of sound waves in air at $20^\circ\text{C} = 343 \text{ m/sec}$.
- Pitch is a change in the frequency of the sound wave.

5.2.2.The Speed of Sound Waves in Various Media

*The speed of any mechanical wave (transverse or longitudinal) depends on both inertial property (to store kinetic energy) and the elastic property (to store potential energy) of the medium.

5.2.2.1.In case of liquids:

Speed of sound waves depends mainly on two factors such as, the compressibility and nature of the medium. This means that:

$$v = \sqrt{\frac{\text{elastic property}}{\text{inertial property}}} = \sqrt{\frac{B}{\rho}} = f \lambda \quad (5.1)$$

Where: B is the bulk modulus of the material and is defined as follow: $B = -P \cdot \frac{V}{\Delta V}$

V is the volume and ΔV is the change in volume.

ρ is the density of the material

f is the frequency

λ is the wavelength

5.2.2.2. In Case of Metallic Rods:

Speed of sound waves can be calculated according to the following equation:

$$V = \sqrt{\frac{Y}{\rho}} = \sqrt{\frac{F \times L}{A \times \Delta L \times \rho}} \quad (5.2)$$

Where:

Y is the Young modulus of the material (F the instantaneous perpendicular force per unit area

(A). the original length (L), ΔL : is the elongation or stretch)

and ρ is the density of the material.

5.2.2.3. In Case of Gasses:

$$V = \sqrt{\frac{\gamma P}{\rho}} \quad (5.3)$$

Where :

γ is a constant and is defined as :

$$\gamma = \frac{C_p}{C_v}$$

Where:

C_p is the heat capacity at constant pressure.

C_v is the heat capacity at constant volume.

P is the pressure and ρ is the density of gas

5.3. Types of Sound Waves:

- 1- Infrasonic waves have a frequency less than 20 Hz.
- 2- Acoustic waves have frequency ranged from 20 Hz o 20000 Hz
- 3- Ultrasonic waves have frequency greater than 20 kHz

Example (5.1)

Calculate the speed of longitudinal waves:

- a) in water; given that the Bulk modulus is $2.1 \times 10^9 \text{ N/m}^2$ and its density is 10^3 kg/m^3 , and
- b) in air at 1 atm; given that the Bulk modulus is $1.41 \times 10^5 \text{ N/m}^2$ and its density is 1.29 kg/m^3 ?

Answer

$$a) \quad V = \sqrt{\frac{B}{\rho}} = \sqrt{\frac{2.1 \times 10^9}{10^3}} \approx 1549 \quad \text{m/s}$$

$$b) \quad V = \sqrt{\frac{B}{\rho}} = \sqrt{\frac{1.41 \times 10^5}{1.29}} \approx 331 \quad \text{m/s}$$

Example (5.2)

If a frequency of 760 kHz is emitted with radio velocity in the order of 3×10^8 m/sec, calculate the wavelength?

Answer

Note that:

$$f = 760 \text{ kHz} = 760 \times 10^3 \text{ Hz}, \quad v = 3 \times 10^8 \text{ m/sec.}$$

$$\text{But, as known: velocity } V = f \cdot \lambda \Rightarrow \text{wavelength is } \lambda = \frac{V}{f} = \frac{3 \times 10^8}{760 \times 10^3} = 395 \text{ m}$$

Example (5.3)

If the speed of sound wave in water is 1530 m/sec and its frequency is 1800 Hz, calculate the wavelength?

Answer

From example, $v = 1530$ m/sec, $f = 1800$ Hz . The velocity is

$$V = f \cdot \lambda \Rightarrow \text{wavelength is } \lambda = \frac{V}{f} = \frac{1530}{1800} = 0.85 \text{ m}$$

Example (5.4)

Suppose that the bulk modulus (B) of water is 2100 Mpa, calculate the speed (v) of sound waves in water? Knowing that, $\rho = 1000 \text{ kg/m}^3$

Answer

Note that:

$$B = 2100 \text{ Mpa} = 2100 \times 10^6 \text{ Pa}, \quad \rho = 1000 \text{ kg/m}^3$$

But, as known:

$$V = \sqrt{\frac{B}{\rho}} \quad V = \sqrt{\frac{B}{\rho}} = \sqrt{\frac{2100 \times 10^6}{1000}} = 1449.14 \text{ m/sec}$$

Example (5.5)

Suppose that the speed (v) of sound waves in steel is 5900 m/sec, calculate the bulk modulus (B) of steel? Knowing that, $\rho = 7900 \text{ kg/m}^3$

Answer

Note that:

$$V = 5900 \text{ m/sec}, \rho = 7900 \text{ kg/m}^3$$

But, as known:

$$V = \sqrt{\frac{B}{\rho}} \quad \Rightarrow$$

$$B = V^2 \times \rho = (5900)^2 \times 7900 \approx 2.75 \times 10^{11} \text{ Pa}$$

Example (5.7)

A dolphin located in sea water at a temperature of 25°C emits a sound directed toward the bottom of the ocean 150 m below. How much time passes before it hears an echo?

If The speed of sound in seawater at 25°C is 1530 m/s.

Answer

The speed of sound in seawater at 25°C is 1530 m/s. Therefore, the time for the sound to reach the sea floor and return is

$$t = \frac{2d}{V} = \frac{2(150 \text{ m})}{1530 \text{ m/s}} = 196 \text{ ms}$$

Example (5.8)

The range of human hearing extends from approximately 20 Hz to 20 000 Hz. Find the wavelengths of these extremes at a temperature of 27°C, knowing that the velocity of the sound at this temperature is 347 m/sec.

Answer

The wavelength of the 20 Hz sound is $\lambda = \frac{V}{f} = \frac{347 \text{ m/s}}{20 \text{ Hz}} = 17 \text{ m},$

and that the wavelength for 20 000 Hz is $\lambda = \frac{347 \text{ m/s}}{20\,000 \text{ Hz}} = 1.7 \times 10^{-2} \text{ m} = 1.7 \text{ cm}.$

Thus, range of wavelengths of audible sounds at 27°C is from 1.7 cm to 17 m.

Example (5.8)

Find the speed of sound in mercury, which has a bulk modulus of approximately $2.80 \times 10^{10} \text{ N/m}^2$ and a density of $13\,600 \text{ kg/m}^3$.

Answer

$$v = \sqrt{\frac{B}{\rho}} = \sqrt{\frac{2.80 \times 10^{10} \text{ N/m}^2}{13\,600 \text{ kg/m}^3}} = 1.43 \times 10^3 \text{ m/s} = 1.43 \text{ Km/s}$$

5.4. The Intensity of Sound Wave (I)

It is the average rate per unit area at which energy is transmitted by the wave .

- The sound intensity;
$$I = \frac{\text{Power}}{4 \pi r^2} = \frac{P}{4 \pi r^2} \quad \text{W/m}^2$$
- The maximum pressure variation;
$$\Delta p_m = \sqrt{2 \rho I V}$$
- Where; I : is the intensity of the sound wave
 ρ : is the density of the medium
 V : is the speed of the wave
 Δp_m : is the change in pressure

N.B. ω : is the angular frequency $\omega = 2\pi f$

Example (5.9)

What is the intensity of the sound waves at distance $r = 2.5$ m from a source if the source emits energy at the rate $P = 25$ J/s ?

- The sound intensity;
$$I = \frac{P}{4 \pi r^2} \quad \text{W/m}^2$$
- The sound intensity;
$$I = \frac{25}{4 \times 3.14 \times (2.5)^2} = 0.318 \quad \text{W/m}^2$$

5.5. The Inverse Square Law

The intensity of waves radiating isotropically from a point source is inversely proportional to the square of the distance from the source. See figure (5.5)

$$\frac{I_1}{I_2} = \left(\frac{r_2}{r_1} \right)^2$$

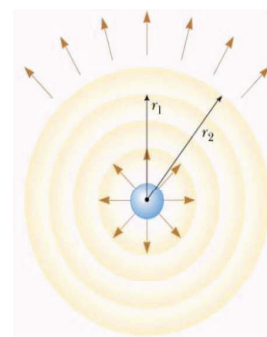


Figure (5.5)

5.6. The Decibel Scale (Intensity Level or Sound Level) (β)

$\beta = 10 \log (I / I_0) \text{ (dB)}$ $\rightarrow$ where, $I_0 = 10^{-2} \text{ (W / m}^2\text{)}$ [Standard Reference Intensity]

Example (5.10)

Find the sound level in decibels of a sound wave that has an intensity of 10^{-5} W/m^2 ?

$$\beta = 10 \log \frac{I}{I_0} = 10 \log \frac{10^{-5}}{10^{-12}} = 70 \text{ dB}$$

Example (5.11)

Find the intensity of a sound wave that has an intensity level of 35 dB ?

$$\beta = 10 \log \frac{I}{I_0}$$

$$35 = 10 \log \frac{I}{10^{-12}} \quad \Rightarrow \quad I = 3.16 \times 10^{-9} \text{ W/m}^2$$

Example (5.12)

The faintest sounds the human ear can detect at a frequency of 1000 Hz correspond to an intensity of about 10^{-12} W/m^2 (the so – called threshold of hearing). Determine the pressure amplitudes associated with these two limits. ($V = 343 \text{ m/s}$ and the density of air to be $\rho = 1.2 \text{ kg/m}^3$) ?

- Since;
$$I = \frac{\Delta p_m^2}{2 \rho V}$$

- - So;
$$\Delta p_m = (2 \rho I)^{1/2} = (2 \times 1.2 \times 343 \times 10^{-12})^{1/2} = 2.87 \times 10^{-5} \text{ N/m}^2$$

Example (5.13)

Two sound waves have intensities I_1 and I_2 . How do their sound levels compare?

-
$$\frac{I_2}{I_1} = \frac{\frac{I_2}{I_0}}{\frac{I_1}{I_0}}$$

- Taking log to both sides;

-
$$\log \left(\frac{I_2}{I_1} \right) = \log \left(\frac{I_2}{I_0} \right) - \log \left(\frac{I_1}{I_0} \right)$$

- So;
$$10 \log \left(\frac{I_2}{I_1} \right) = 10 \log \left(\frac{I_2}{I_0} \right) - 10 \log \left(\frac{I_1}{I_0} \right)$$

- So;
$$10 \log \left(\frac{I_2}{I_1} \right) = \beta_2 - \beta_1$$

Example (5.14)

The sound emitted by a source reaches a particular position with an intensity I_1 , what is the change in intensity level when another identical source is placed with the first ?

- Since; $\beta_1 = 10 \log \left(\frac{I_1}{I_o} \right)$; $\beta_2 = 10 \log \left(\frac{I_2}{I_o} \right)$
- And; $I_2 = I_1 + I_1 = 2 I_1$
- So; $\beta_2 - \beta_1 = 10 \log \left(\frac{I_2}{I_1} \right) = 10 \log \left(\frac{2 I_1}{I_1} \right) = 3 \text{ dB}$

Example (5.15)

A crying child delivers about 1 mW of power, (a) If this power is uniformly distributed in all directions, what is the sound intensity level at a distance of 5 m ?

(b) What would be the intensity level of two children crying at the same time if each delivered 1 mW of power?

- Since; $I_1 = \frac{P}{4 \pi r^2} = \frac{1 \times 10^{-3}}{4 \times 3.14 \times (5)^2} = 3.183 \times 10^{-6} \text{ W/m}^2$
- So; $\beta = 10 \log \frac{I_1}{I_o} = 10 \log \frac{3.183 \times 10^{-6}}{10^{-12}} = 65.03 \text{ dB}$
- When another one crying; $I = I_1 + I_1 = 2 I_1 = 6.366 \times 10^{-6} \text{ W/m}^2$
- So; $\beta = 10 \log \frac{I}{I_o} = 10 \log \frac{6.366 \times 10^{-6}}{10^{-12}} = 68.04 \text{ dB}$

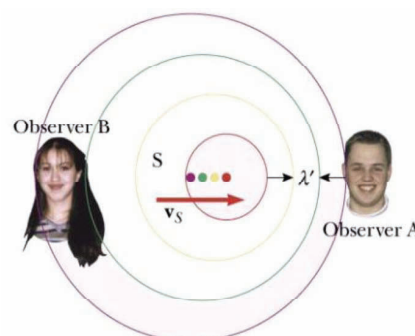
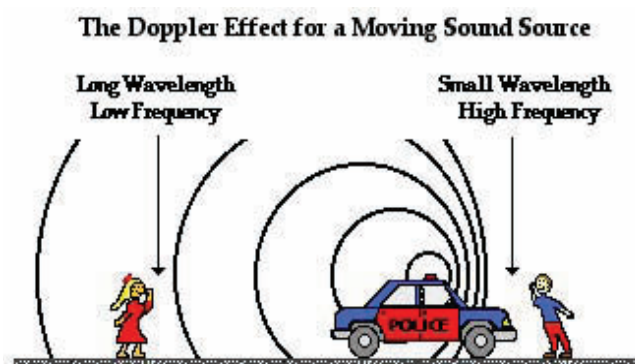
5.7. The Doppler Effect, See figure(5.6)

Figure (5.6)

It is a phenomenon in which the measured frequency (f') is different from the source frequency (f) because of the relative motion between them.

*Assume; V : speed of wave. V_o : speed of observer; V_s : speed of source

5.7.1. Observer moving and Source at rest

$$f' = f \left(\frac{V \pm V_o}{V} \right) \quad (+) \Rightarrow \text{approach, toward} \quad (-) \Rightarrow \text{apart, away from}$$

- 1- The frequency heard by the observer, f' appears higher when the observer

approaches the source $f' = f \left(\frac{V + V_o}{V} \right)$

- 2- The frequency heard by the observer, f' , appears lower when the observer moves

away from the source $f' = f \left(\frac{V - V_o}{V} \right)$

5.7.2. Source moving and Observer at rest

$$f' = f \left(\frac{V}{V \pm V_s} \right) \quad (-) \Rightarrow \text{approach, toward} \quad (+) \Rightarrow \text{apart, away from}$$

- 1- When the source is moving toward the observer, the apparent frequency is higher

$$f' = f \left(\frac{V}{V - V_s} \right)$$

- 2- When the source is moving away from the observer, the apparent frequency is

lower $f' = f \left(\frac{V}{V + V_s} \right)$

5.7.3. Both Observer and Source moving

$$f' = f \left(\frac{V \pm V_o}{V \pm V_s} \right) \quad \left(\begin{smallmatrix} + \\ - \end{smallmatrix} \right) \Rightarrow \text{approach, toward} \quad \left(\begin{smallmatrix} - \\ + \end{smallmatrix} \right) \Rightarrow \text{apart, away from}$$

$f' = f \left(\frac{V + V_o}{V - V_s} \right)$	$f' = f \left(\frac{V - V_o}{V + V_s} \right)$
$f' = f \left(\frac{V - V_o}{V - V_s} \right)$	$f' = f \left(\frac{V + V_o}{V + V_s} \right)$

Example (5.16)

A train moving at a speed of 40 m/s sounding its whistle, which has a frequency of 500 Hz. Determine the frequency heard by a stationary observer as a train approaches and then recedes from the observer. Take $V = 343$ m/s for the speed of sound in air ?

$$V_s = 40 \text{ m/s} \quad V_o = 0 \text{ m/s} \quad V = 343 \text{ m/s} \quad f = 500 \text{ Hz}$$

- As the train approach;

$$f' = f \left(\frac{V}{V - V_s} \right) = 500 \times \left(\frac{343}{343 - 40} \right) = 566 \text{ Hz}$$

- As the train recedes;

$$f' = f \left(\frac{V}{V + V_s} \right) = 500 \times \left(\frac{343}{343 + 40} \right) = 447.8 \text{ Hz}$$

Example (5.17)

An ambulance travels down a highway at a speed of 33.5 m/s. Its siren emits sound at a frequency of 400 Hz. What is the frequency heard by a passenger in a car traveling at 24.6 m/s in the opposite direction as the car approaches the ambulance and as the car moves away from the ambulance ?

$$V_s = 33.5 \text{ m/s} \quad V_o = 24.6 \text{ m/s} \quad V = 343 \text{ m/s} \quad f = 400 \text{ Hz}$$

$f' = f \left(\frac{V + V_o}{V - V_s} \right) = 400 \times \left(\frac{343 + 24.6}{343 - 33.5} \right) = 475.1 \text{ Hz}$	$f' = f \left(\frac{V - V_o}{V + V_s} \right) = 400 \times \left(\frac{343 - 24.6}{343 + 33.5} \right) = 375.1 \text{ Hz}$
--	--

- As the car moves away from the ambulance;



$$f' = f \left(\frac{V - V_o}{V + V_s} \right) = 400 \times \left(\frac{343 - 24.6}{343 + 33.5} \right) = 338.3 \text{ Hz}$$

Example (5.18)

A stationary source emits a whistle at a frequency of 200 Hz. If the velocity of propagation of the sound wave is 343 m/s, find the observed frequency if (a) the observer is approaching the source at 25 m/s and (b) the observer is receding from the source at 25 m/s.

$$V_s = 0 \text{ m/s} \quad V_o = 25 \text{ m/s} \quad V = 343 \text{ m/s} \quad f = 200 \text{ Hz}$$

- As the observer approach;

$$f' = f \left(\frac{V + V_o}{V} \right) = 200 \times \left(\frac{343 + 25}{343} \right) = 214.6 \text{ Hz}$$

- As the observer recedes;

$$f' = f \left(\frac{V - V_o}{V} \right) = 200 \times \left(\frac{343 - 25}{343} \right) = 185.4 \text{ Hz}$$

Example (5.19)

A researcher notices that the frequency of a note emitted by an automobile horn appears to drop from 284 cycles/s to 266 cycles/s as the automobile passes him. Calculate the speed of the car, Take $V = 1100 \text{ ft/s}$ as the speed of sound in air ?

$$V_o = 0 \text{ ft/s} \quad V = 1100 \text{ ft/s} \quad f = 284 \text{ Hz} \quad f' = 266 \text{ Hz}$$

- As the automobile passes the researcher;

$$f' = f \left(\frac{V}{V + V_s} \right) \Rightarrow 266 = 284 \times \left(\frac{1100}{1100 + V} \right) \Rightarrow V_s = 74.44 \text{ ft/s}$$

Example (5.20)

An airplane is flying at Mach 0.5 and carries a sound source that emits a 1000 Hz signal. What frequency sound does a listener hear if he is in the path of the airplane after the airplane has passed?

- **Note That;** **Mach = the speed of the sound in air**

$$V_o = 0 \text{ m/s} \quad V = 343 \text{ m/s} \quad V_s = 0.5 \text{ Mach} = 0.5 \times 343 = 171.5 \text{ m/s}$$

$$f = 1000 \text{ Hz} \quad f' = ?$$

- As the automobile passes the researcher;

$$f' = f \left(\frac{V}{V + V_s} \right) = 1000 \times \left(\frac{343}{343 + 171.5} \right) = 666.7 \text{ Hz}$$

5.8. Standing Waves in Air Columns

- The standing waves in the air tubes (such as the argon machine) are formed as a result of interference between the longitudinal waves that move in opposite directions.
- The relationship between the fallen wave and the reflected wave is dependent on the state of the tube (is it open from both sides or is it open from one side)

5.7.1. Open at Both Ends See figure(5.7)

- The two waves (the incident wave and the reflected wave) is nearly in phase.
- The wavelength is twice the length of the pipe.

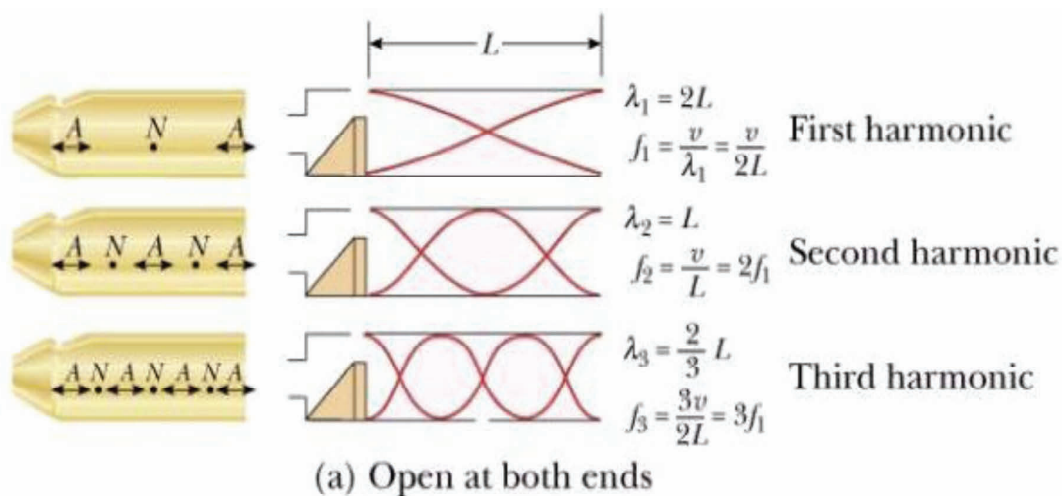


Figure (5.7)

$$\text{So; } f_n = \frac{n}{2L} V \quad ; \quad f_n = n f_1 \quad n = 1, 2, 3, \dots$$

f_1 : is the fundamental frequency

f_2 : is the second harmonic

f_3 : is the third harmonic

f_n : is the n^{th} harmonic

L : length of the pipe

$$f_2 = 2 f_1$$

$$f_3 = 3 f_1$$

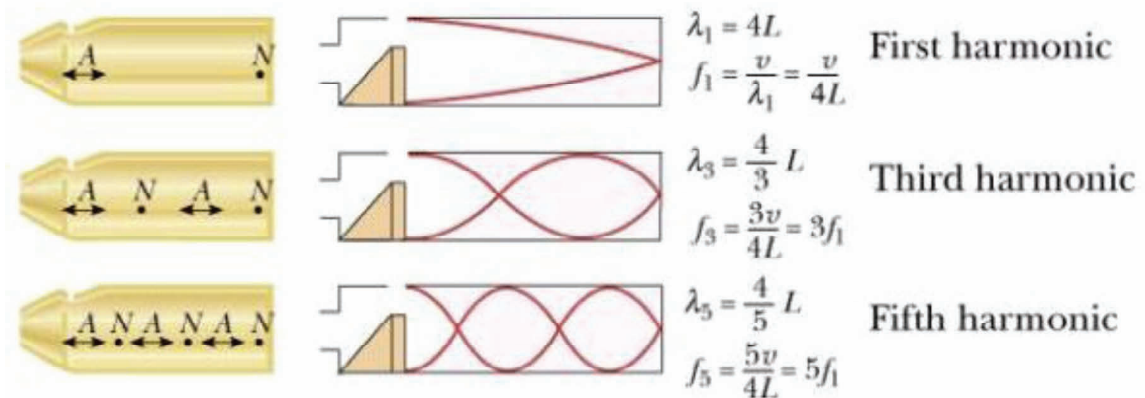
$$f_n = n f_1$$

Note:

“In a pipe open at both ends; the natural frequencies of vibration form a harmonic series, that is, the higher harmonics are integral multiples of the fundamental frequency”

5.7.2. Closed at One End, See figure (5.8)

- The reflected wave is 180° out of phase with the incident wave .
- The wavelength is four time the length of the pipe.



(b) Closed at one end, open at the other

Figure (5.8)

- So; $f_n = f_n = \frac{n}{4L} V$; $f_n = n f_1$ $n = 1, 3, 5, \dots$

..

Note:

“In a pipe closed at one end; only odd harmonics are present ”

Example (5.21):

- Determine the wavelength λ , the period T , the frequency f , the amplitude A and the speed V see figure(5.9) and (5.10)

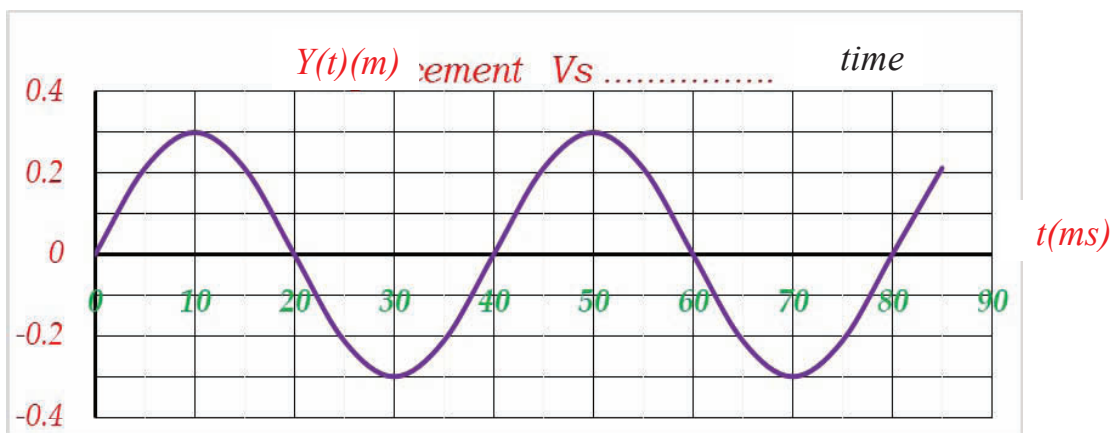


Figure (5.9)

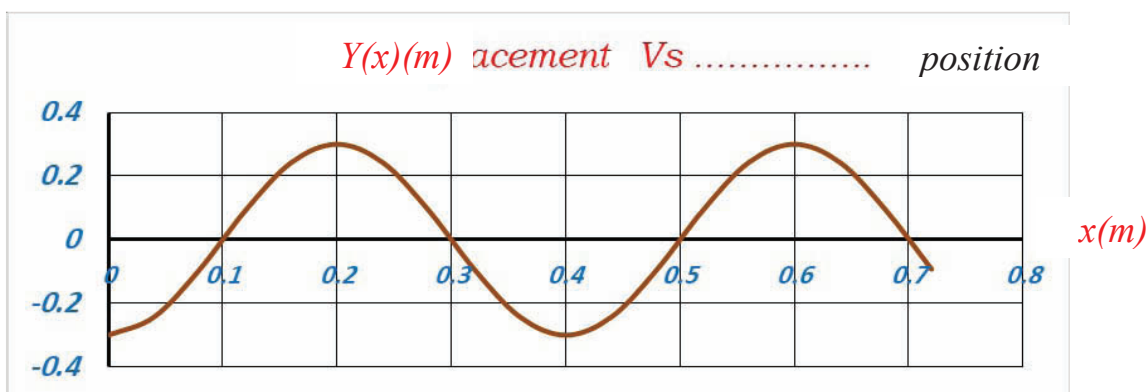


Figure (5.10)

From "Displacement Vs displacement" we read $\lambda = 0.4m$

From "Displacement Vs time" we read $T = 40ms$

From the two curves, we read $A = 0.3m$

With these results, we can calculate:

$$f = \frac{1}{T} = \frac{1}{40 \times 10^{-3}} = 25Hz$$

$$V = \lambda \times f = 0.4 \times 25 = 10ms$$

5.9. Problems**Choose The Correct Answer in Each of The Followings:**

- 1) Infrasonic waves have a frequency less than:
A) 10 Hz
B) 2 Hz
C) 20 Hz
D) 50 Hz
- 2) The speed of a sound wave is determined by:
A) its amplitude
B) its intensity
C) number of harmonics present
D) the transmitting medium
- 3) A sound wave has a wavelength of 3.0m. The distance from a compression center to the adjacent rarefaction center is:
A) 0.75m
B) 1.5m
C) 3.0m
D) need to know wave speed
- 4) The sound intensity 5.0m from a point source is 0.50W/m^2 . The power output of the source is:
A) 39W
B) 160W
C) 266W
D) 320W
- 5) The standard reference sound level is about:
A) The threshold of human hearing at 1000 Hz
B) The threshold of pain for human hearing at 1000 Hz
C) The level of sound produced when the 1 kg standard mass is dropped 1m onto a concrete floor
D) The level of normal conversation
- 6) The intensity of sound wave A is 100 times that of sound wave B. Relative to wave B the sound level of wave A is:
A) -2 dB
B) +2 dB
C) +10 dB
D) +20 dB
- 7) When a sine wave is used to represent a sound wave, the crest corresponds to:
A) Rarefaction
B) Condensation
C) Point where molecules vibrate at a right angle to the direction of wave travel
D) Region of low elasticity

- 8) A sound wave coming from a tuba has a wavelength of 1.50 m and travels to your ears at a speed of 345 m/s. What is the frequency of the sound you hear?
- A) 517 Hz
 - B) 1/517 HZ
 - C) 230 Hz
 - D) 1/230 Hz
- 9) A series of ocean waves, 5.0 m between crests, moves past at 2.0 waves/s. Find their speed.
- A) 2.5 m/s
 - B) 5.0 m/s
 - C) 8.0 m/s
 - D) 10 m/s
- 10) If the tension on a guitar string is increased by a factor of 3, the fundamental frequency at which it vibrates is changed by what factor?
- A) 9
 - B) 3
 - C) $\sqrt{3}$
 - D) $\frac{1}{\sqrt{3}}$
- 11) What phenomenon is created by two tuning forks, side-by-side, emitting frequencies, which differ by only a small amount?
- A) Resonance
 - B) Interference
 - C) The Doppler Effect
 - D) Beats
- 12) The..... is defined as the number of cycles of a periodic wave occurring per unit time.
- A) wavelength
 - B) Frequency
 - C) Amplitude
 - D) Period
- 13) Many wave properties are dependent upon other wave properties. Yet, one wave property is independent of all other wave properties. Which one of the following properties of a wave is independent of all the others?
- A) wavelength
 - B) Frequency
 - C) Velocity
 - D) Period

14) Consider the diagram below of several circular waves created at various times and locations. The figure (5.11) (diagram) illustrates

- A) Interference
- B) Diffraction
- C) The Doppler Effect.
- D) Polarization

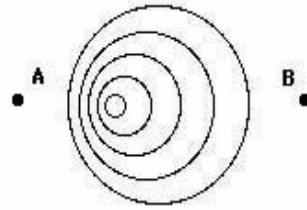


Figure (5.11)

15) The time required for the sound waves ($v = 340 \text{ m/s}$) to travel from the tuning fork to point A (in second) Is.....see figure(5.12)

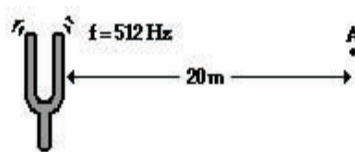


Figure (5.12)

- A) 0.020
- B) 0.059
- C) 0.590
- D) 2.900

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Chapter (6)

Kinematics and Vectors



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Chapter (6)

Kinematics and Vectors

6.1. Mechanics Deals with The Motion of Objects

1. What specifies the motion?
2. Where is it located?
3. When was it there?
4. How fast is it moving?

■ Mechanics of the broad science concerned with the movement of objects and their causes, and the ramifications of this other branches of science such as kinematics and Dynamics. Kinematics cares science as the movement of objects without regard to their causes; while aware of the dynamics it studies the movement of objects and their causes such as strength and mass.

The motion is define as a continuous change in the position of an object.

6.2. Type of Motion:

- 1) **Translational motion:** such as (car moving down a highway)
- 2) **Rotational motion:** such as (Earth's spin on its)
- 3) **vibration motion:** such as (back-and-forth movement of a pendulum)

6.3. Translational Motion:

In general, suppose that a particle is a point-like mass having infinitesimal size respect to the distance moving.

6.3.1 Define Vector and Scalar:

Vector is defined as a physical quantity that requires the specification of both direction and magnitude. Such as (velocity, force, displacement) as shown figure (6.1).



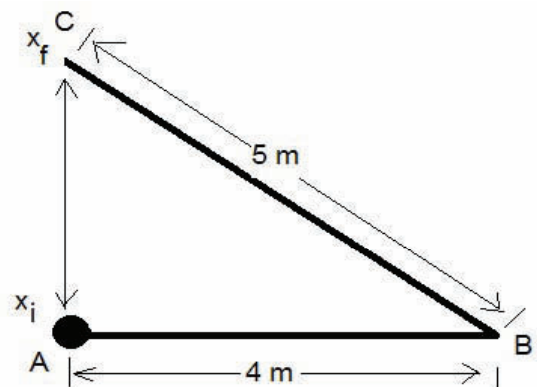
Scalar is defined as a quantity that has magnitude and no direction.

Such as (mass, ., distance,.....)

Example (6.1)

Calculate the distance and displacement traveled the particle which show in the following figure (6.2)?

Figure (6.2)



Solution:

$$\text{Distance } x = ABC = 4\text{ m} + 5\text{ m} = 9\text{ m}$$

$$\text{Displacement } \Delta x = x_f - x_i = AC$$

$$(CB)^2 = (AB)^2 + (AC)^2$$

$$(AC)^2 = (CB)^2 - (AB)^2 = 25 - 16 = 9$$

$$\Delta x = AC = 3\text{ m}$$

6.3.2. The Properties of Vectors.**6.3.2.1. Equality of Two Vectors**

Two vectors A and B are equal if they have the same magnitude and the same direction regardless of the position of their initial points as in Figure (6.3).

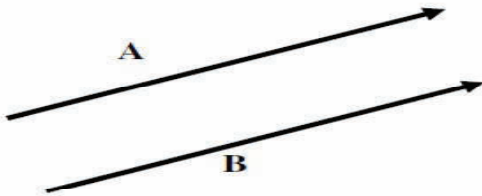


Figure (6.3)

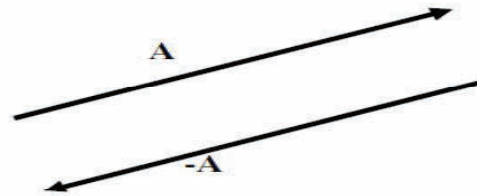


Figure (6.4)

6.3.2.2. The Negative of a Vector

The negative of a vector A is another vector having the same magnitude as A but its direction is opposite to A, as in Figure (6.4). It is denoted by (-A).

6.3.2.3. The Sum or The Resultant of Two Vectors A and B

is another vector C. To obtain the vector C geometrically we have two rules.

6.3.2.3.1. Triangle Rule.

From, Figure (6.5) and Figure (6.6) when draw a line from the free tail of B to the free head of A to construct the vector C

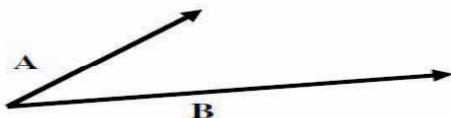


Figure (6.5)



Figure (6.6)

6.3.2.3.2. Parallelogram Rule

A and B are drawn as two sides of a parallelogram, then C is its diagonal, see Figure (6.7)

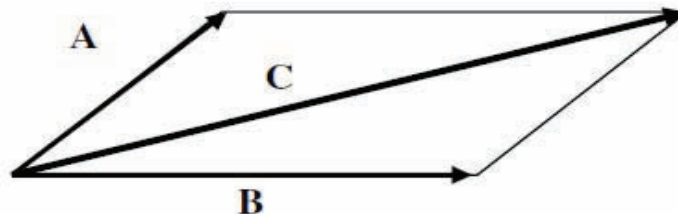


Figure (6.7)

6.3.2.3.3. The Polygon Rule

The sum or the resultant of a group of vectors **A**, **B**, **C**, and **D** is a vector **E** as shown Figure (6.8)

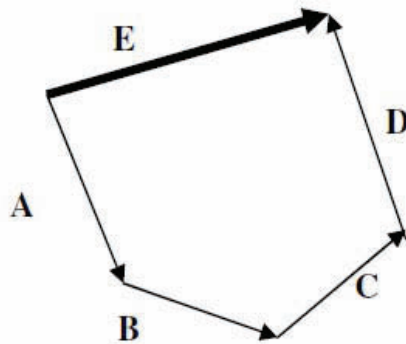


Figure (6.8)

6.3.2.4. The Difference of Vectors

The difference of vectors **A** and **B** represented by **A-B** is the sum of vector **A** and the negative of vector **B**. $C = A + (-B)$, see Figure (6.9)

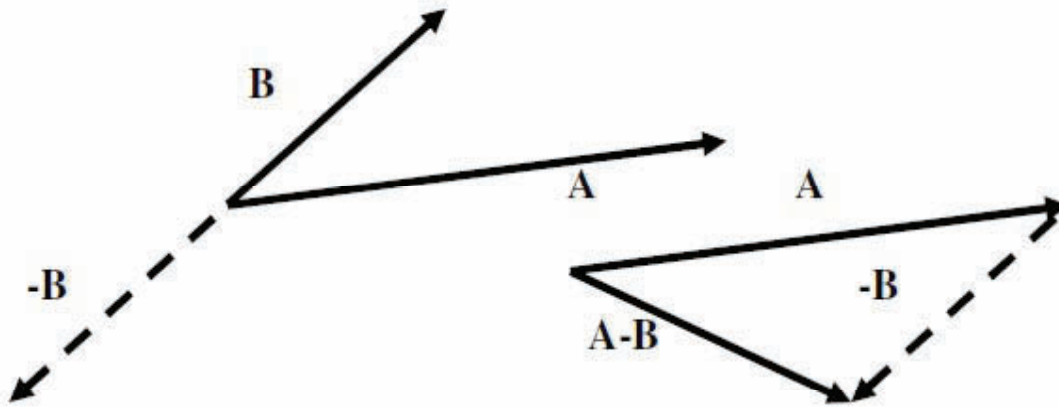


Figure (6.9)

6.3.3. The Mathematical Representation for Vectors

1- Vector Addition of $\vec{A}$ and $\vec{B}$ in tow dimensions is

See figure (6.10)

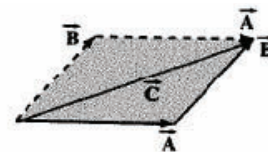


Figure (6.10)

$$\vec{C} = \vec{A} + \vec{B} = (A_x + B_x)\vec{i} + (A_y + B_y)\vec{j}$$

2- The subtraction of $\vec{A}$ and $\vec{B}$ in tow dimensions is

See figure (6.11)

$$\vec{C} = \vec{A} - \vec{B} = (A_x - B_x)\vec{i} + (A_y - B_y)\vec{j}$$

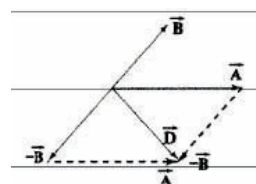


Figure (6.11)

Any vector in three dimensions $\vec{A} = A_x \vec{i} + A_y \vec{j} + A_z \vec{k}$ and. $\vec{B} = B_x \vec{i} + B_y \vec{j} + B_z \vec{k}$

3-Addition of two vectors:

$$\vec{C} = \vec{A} + \vec{B} = (A_x + B_x) \vec{i} + (A_y + B_y) \vec{j} + (A_z + B_z) \vec{k}$$

4- Vector Product

4-1 Scalar (Dot) double product

Also called (Parallel Product) $\vec{A} \cdot \vec{B}$

$$\vec{A} \cdot \vec{B} = AB \cos(\theta) = A_x B_x + A_y B_y + A_z B_z$$

$$\text{Also } \vec{A} \cdot \vec{B} = |\vec{A}| |\vec{B}| \cos(\theta)$$

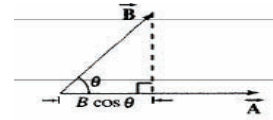


figure (6.12)

It is also referred to as the projection of $\vec{A}$ on $\vec{B}$, or vice versa.

See figure (6.12)

Where

- θ is the direction between A and B $\cos(\theta) = \frac{\vec{A} \cdot \vec{B}}{|\vec{A}| |\vec{B}|} = \frac{A_x B_x + A_y B_y + A_z B_z}{|\vec{A}| |\vec{B}|}$

- $|\vec{A}|$ and $|\vec{B}|$ are the amplitude of the two vectors

$$|\vec{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2} = \sqrt{\vec{A} \cdot \vec{A}} \text{ and } |\vec{B}| = \sqrt{B_x^2 + B_y^2 + B_z^2} = \sqrt{\vec{B} \cdot \vec{B}}$$

4-2 Vector (Cross) double product (perpendicular product $\vec{A} \perp \vec{B}$)

$$\vec{C} = \vec{A} \times \vec{B} = AB \sin(\theta) = \text{area of Parallelogram see figure (6.13)}$$

$$\vec{C} = \vec{A} \times \vec{B} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = (A_y B_z - A_z B_y) \vec{i} + (A_z B_x - A_x B_z) \vec{j} + (A_x B_y - A_y B_x) \vec{k}$$

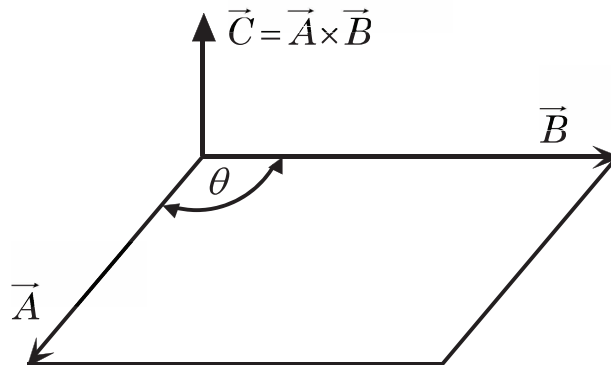


Figure (6.13)

4-3 Scalar triple product

$$\vec{A} \cdot (\vec{B} \times \vec{C}) = \begin{vmatrix} A_x & A_y & A_z \\ B_x & B_y & B_z \\ C_x & C_y & C_z \end{vmatrix} =$$

$$\vec{A} \cdot (\vec{B} \times \vec{C}) = A_x B_y C_z + A_y B_z C_x + A_z B_x C_y - A_x B_z C_y - A_y B_x C_z - A_z B_y C_x$$

Also, $\vec{A} \cdot (\vec{B} \times \vec{C}) = \vec{C} \cdot (\vec{A} \times \vec{B}) = \vec{B} \cdot (\vec{C} \times \vec{A})$

Note:

$$\vec{i} \times \vec{i} = \vec{j} \times \vec{j} = \vec{k} \times \vec{k} = 0$$

$$\vec{i} \times \vec{j} = \vec{k} = -\vec{j} \times \vec{i}, \vec{j} \times \vec{k} = -\vec{k} \times \vec{j} = \vec{i}, \vec{k} \times \vec{i} = -\vec{i} \times \vec{k} = \vec{j}$$

4-4 Vector triple product $\vec{A} \perp \vec{B}$

$$\vec{A} = A_x \vec{i} + A_y \vec{j} + A_z \vec{k}, \quad \vec{B} = B_x \vec{i} \quad \text{and} \quad \vec{C} = C_x \vec{i} + C_y \vec{j}.$$

Therefore, $\vec{B} \times \vec{C} = B_x C_y \vec{k}$

$$\begin{aligned} \underline{A} \times (\underline{B} \times \underline{C}) &= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ A_x & A_y & A_z \\ 0 & 0 & B_x C_y \end{vmatrix} = A_y B_x C_y \vec{i} - A_x B_x C_y \vec{j} \\ &= A_y B_x C_y \vec{i} + A_x B_x C_y \vec{i} - A_x B_x C_y \vec{j} - A_x B_x C_y \vec{j} \\ &= B_x \vec{i} (A_y C_y + A_x C_x) - A_x B_x (C_x \vec{i} + C_y \vec{j}) \end{aligned}$$

So that

$$\vec{A} \times (\vec{B} \times \vec{C}) = \vec{B} \cdot (\vec{A} \vec{C}) - \vec{C} \cdot (\vec{A} \vec{B})$$

Example (6.2)

We have $\vec{A} = 2\vec{i} + 4\vec{j}$ & $\vec{B} = 5\vec{i} + 2\vec{j}$, See figure (6.14)

Calculate the $\vec{A} + \vec{B}$ & $\vec{A} - \vec{B}$

$$\vec{A} = 2\vec{i} + 4\vec{j} \text{ \& } \vec{B} = 5\vec{i} + 2\vec{j} \Rightarrow \vec{A} + \vec{B} = (2+5)\vec{i} + (4+2)\vec{j} = 7\vec{i} + 6\vec{j}$$

$$\vec{A} - \vec{B} = (2-5)\vec{i} + (4-2)\vec{j} = -3\vec{i} + 2\vec{j}$$

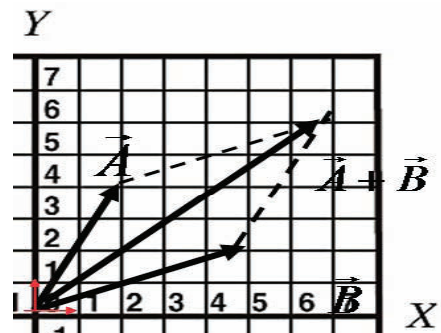


Figure (6.14)

Example (6.3)

We have $\vec{A} = 2\vec{i} + 4\vec{j}$ & $\vec{B} = 5\vec{i} + 2\vec{j}$

Calculate the angle $\vec{A}, \vec{B}$ between θ

$$\vec{A} = 2\vec{i} + 4\vec{j} \text{ & } \vec{B} = 5\vec{i} + 2\vec{j}$$

$$\vec{A} \cdot \vec{B} = (2)(5) + (4)(2) = 18$$

$$|A| = \sqrt{A_x^2 + A_y^2} = \sqrt{2^2 + 4^2} = \sqrt{20} \text{ and}$$

$$|B| = \sqrt{B_x^2 + B_y^2} = \sqrt{5^2 + 2^2} = \sqrt{29}$$

$$\text{also } \vec{A} \cdot \vec{B} = |A||B|\cos(\theta) \text{ & } \cos\theta = \frac{18}{\sqrt{20} \cdot \sqrt{29}} = 0.748 \Rightarrow$$

$$\theta = 41.63^\circ$$

Example (6.4)

We have $\vec{A} = 2\vec{i} + 4\vec{j}$ & $\vec{B} = 5\vec{i} + 2\vec{j}$

Calculate the $|\vec{A} + \vec{B}|$ & $|\vec{A} - \vec{B}|$

$$\vec{A} = 2\vec{i} + 4\vec{j} \text{ & } \vec{B} = 5\vec{i} + 2\vec{j} \Rightarrow \vec{A} + \vec{B} = (2+5)\vec{i} + (4+2)\vec{j} = 7\vec{i} + 6\vec{j}$$

$$|\vec{A} + \vec{B}| = \sqrt{7^2 + 6^2} = \sqrt{85}$$

$$\vec{A} - \vec{B} = (2-5)\vec{i} + (4-2)\vec{j} = -3\vec{i} + 2\vec{j}$$

$$|\vec{A} - \vec{B}| = \sqrt{(-3)^2 + (2)^2} = \sqrt{13}$$

6.4. Kinematics of Motion

The position of an object along a straight line can be uniquely identified by its distance from a (user chosen) origin x_i . (See Figure 6.15).

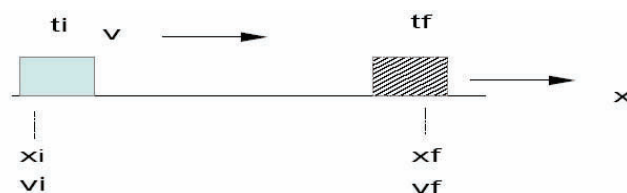


Figure (6.15)

The motion in one dimension i.e in x -direction

6.4.1. Displacement :

Displacement vector which represent the change in the position vector

$$\Delta x = dx = x_f - x_i \quad (6.1)$$

Where, x_i is the initial point , and x_f is the final point

6.4.2. Velocity v :

The **average velocity** $\bar{V}$ of an object is defined as the ratio of the displacement to the time interval: see Figure (6.16)

$$\bar{V} = V_{av} = \frac{x_f - x_i}{t_f - t_i} = \frac{\Delta x}{\Delta t} = V_f - V_i \quad (6.2)$$

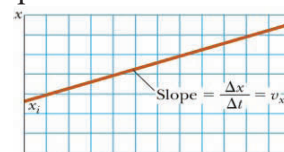


Figure (6.16)

- The x indicates motion along the x -axis
- Is also the slope of the line in the position – time graph

Where, x_i is the initial displacement and x_f is the final displacement. t_i is the initial time t_f is the final time

The **instantaneous velocity** of a particle is defined as the limit of the average velocity as the time interval approaches zero.

$$V = \lim_{\Delta t \rightarrow 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt} \quad (6.3)$$

The unit of the velocity is (m/s)

Average Speed

- Speed is a scalar quantity.
- Average Speed= total distance / total time.
- The speed has no direction and is always expressed as a positive number

6.4.3. Acceleration “a” :

The **average acceleration of a particle** is defined as the ratio of the change in the instantaneous velocity to the time interval. see Figure (6.17)

$$a = \frac{V_f - V_i}{t_f - t_i} = \frac{\Delta V}{\Delta t} = a_f - a_i \quad (6.4)$$

Where, v_i is the initial velocity and v_f is the final velocity

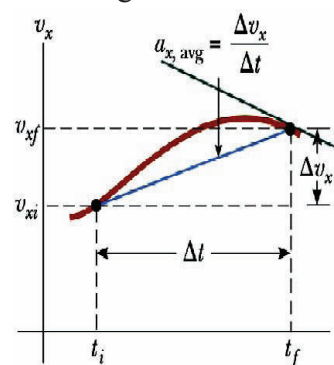


Figure (6.17)

- In one dimension, positive and negative can be used to indicate *direction*

The instantaneous acceleration is defined as the limiting value of the ratio of the average velocity to the time interval as the time approaches zero.

$$a = \lim_{\Delta t \rightarrow 0} \frac{\Delta V}{\Delta t} = \frac{dV}{dt} \quad \text{and for } x\text{-direction } a_x = \frac{d^2x}{dt^2} \quad (6.5)$$

The Instantaneous acceleration equal slope of drawing tangent line of curve at the instantaneous time t

Example (6.5)

a particle moves with the x coordinate equation given by: $x = 3t^3 + 4t$
Find its velocity and acceleration.

Solution:

$$x = 3t^3 + 4t \quad (1)$$

The velocity of the particle can be found

$$\text{as } V = \frac{dx}{dt} = 9t^2 + 4 \quad m/s \quad (2)$$

the acceleration of the particle can be found by differentiating eq. (2)

$$a = \frac{dV}{dt} = 18t \quad m/s^2$$

Example (6.6)

a particle moves with the x coordinate equation given by:

$$x(t) = 7.8 + 9.2t - 2.1t^3$$

Find its velocity and acceleration and velocity of the object at ($t = 3.5\text{sec}$)

Solution:

Velocity is given by

$$V = \frac{dx}{dt} = \frac{d}{dt}(7.8 + 9.2t - 2.1t^3) = 9.2 - 6.3t^2 \quad m/s$$

$$\text{Acceleration is given by } a = \frac{dV}{dt} = \frac{d}{dt}(9.2 - 6.3t^2) = -6.3 \times 2t = -12.6t \quad m/s^2$$

The velocity of the object at ($t = 3.5\text{ s}$) can now be calculated:

$$V = 9.2 - 6.3t^2 = 9.2 - 6.3(3.5)^2 \approx 68 \quad m/s$$

$$a = -6.3 \times 2t = -12.6t = -12.6 \times (3.5) = -44.1 \quad m/s^2$$

Example (6.7)

Find the acceleration from a time-velocity graph see Figure (6.18)

Solution

$$a = \frac{dV}{dt} = \frac{0}{3-0} = 0 \quad m/s^2$$

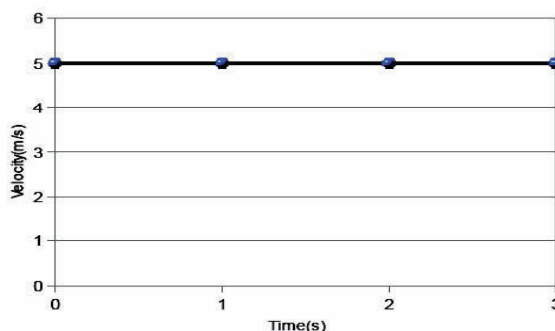


Figure (6.18)

6.5. Kinematic Equations

- The kinematic equations can be used with any particle under uniform acceleration.
- may be used to solve any problem involving one-dimensional motion with a constant acceleration
- You may need to use two of the equations to solve one problem

6.5.1. Motion at Constant Velocity

Suppose particle moves along a straight line, which we will use as the x-axis.

Then for constant velocity, Can we obtain $x(t)$ from v_x ?

See Figure (6.19)

$$V_x = \frac{dx}{dt} = c \quad (6.6)$$

by integration equation (6.6) we have

$$\int_{x_0}^x dx = \int_0^t V_x dt \Rightarrow x = x_0 + V_x t \Rightarrow x_f = x_i + V t \quad (6.7)$$

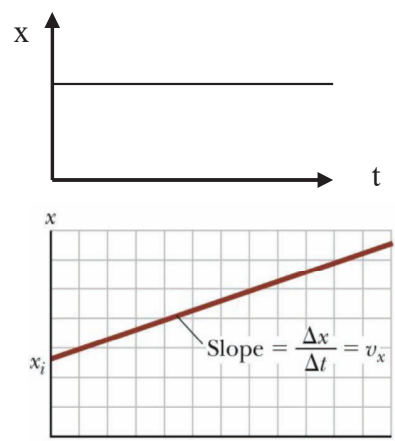


Figure (6.19)

6.5.2. Motion with Constant Acceleration

Let's assume our motion is along the x axis, and represents the x component of the acceleration.

Then for constant acceleration, Can we obtain V_x and $x(t)$ from a_x ? See figure (6.20)

$$a_x = \frac{dV_x}{dt} = c \quad (6.8)$$

By integration equation (6.8) we obtain v_x

$$\int_{v_0}^v dv_x = a_x \int_0^t dt \Rightarrow V_x = V_0 + a_x t \quad \text{or} \quad V_f = V_i + a_x t \quad (6.9)$$

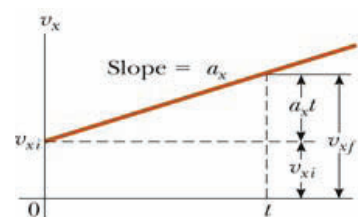


Figure (6.20)

Here, V_0 is the initial velocity must be known in the calculation

By integration equation (6.9) we obtain $x(t)$

$$dx = V_x dt = (V_0 + a_x t) dt \Rightarrow$$

$$\int_{x_0}^x dx = \int_0^t (V_0 + a_x t) dt \Rightarrow x - x_0 = V_0 t + \frac{1}{2} a_x t^2 \text{ or}$$

$$x_f = x_i + V_{0i} t + \frac{1}{2} a_x t^2 \quad (6.10)$$

Example (6.8)

A body starts its motion at 5 m from origin and end its motion at 30 m from origin calculate its velocity if it takes 4 sec.

Solution: The final point can be calculated as:

$$x_f = x_i + Vt \Rightarrow 30 = 5 + V \times 4 \Rightarrow 25 = 4V \Rightarrow V = 6.25 \quad m/s$$

Which mean that the body will reach a point at 30 m far from the origin after 4 sec. At velocity 6.25 m/s

Example (6.9)

A car starting from rest attains a speed of 28 m/sec in 20 sec. Find the acceleration of the car and the distance it travels in this time

Solution

$$a_x = \frac{V_f - V_i}{t} = \frac{28 - 0}{20} = 1.4 \text{ m/sec}^2$$

$$\Delta x = (x_f - x_i) = V_{0i} t + \frac{1}{2} a_x t^2 = (0 \times 20) + \frac{1}{2} (1.4)(20)^2 = 280 \text{ m}$$

Example (6.10)

A bicyclist accelerates at a rate of 4 ft/sec². If she starts at rest and accelerates at this rate for 5 seconds, what is her final velocity and what distance does she go?

Solution

$$V_f = V_i + a_x t = 0 + (4 \times 5) = 20 \text{ ft/sec}$$

$$\Delta x = (x_f - x_i) = V_{0i} t + \frac{1}{2} a_x t^2 = (0 \times 5) + \frac{1}{2} (4)(5)^2 = 50 \text{ ft}$$

Example (6.11)

How long does it take a car going 30 m/sec to stop if it decelerates at 7 m/sec²?

Solution

$$V_f = V_i + a_x t \Rightarrow$$

$$t = \frac{V_f - V_i}{a} = \frac{0 - 30}{-7} = 4.3 \text{ sec}$$

$$x_f = x_i + V_i t + \frac{1}{2} a_x t^2 \Rightarrow x_f = 0 + (30 \times 4.3) + \left(\frac{1}{2}\right)(-7)(4.3)^2 = 64.29 \text{ m}$$

6.5.3. Free Fall

Free fall i.e. Motion of an object moves only under the influence of gravitational acceleration g (9.8 m/sec^2). The Kinematics equation become

$$V = V_0 + gt$$

$$x = x_0 + V_0 t + \frac{1}{2} g t^2$$

$$V^2 = V_0^2 + 2gx \quad (6.11)$$

Galileo's (1564) experiments produced a surprising result .

All objects fall with the same acceleration regardless of mass and shape. See figure (6.21)
 $g = 9.8 \text{ m/s}^2$ or 32 ft/s^2 neglecting air resistance

$$V = V_0 - gt$$

$$y = y_0 + \frac{1}{2} (V + V_0) gt$$

$$y = y_0 + V_0 t - \frac{1}{2} g t^2$$

$$V^2 = V_0^2 + 2g(y - y_0) \quad (6.12)$$

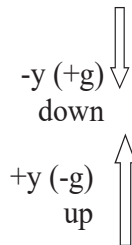


Figure (6.21)

Example (6.12)

A stone is dropped from rest from the top of a building. After 3sec of free fall, what is the displacement y of the stone?

Solution

From equation

$$y = y_0 + V_0 t - \frac{1}{2} g t^2$$

$$y = 0 + 0 - \frac{1}{2} \times (9.8)(3)^2 = -44.1 \text{ m}$$

Example (6.13)

A stone is thrown upwards from the edge of a cliff 18m high as shown in Figure (6.22). It just misses the cliff on the way down and hits the ground below with a speed of 18.8m/s.

- (a) With what velocity was it released?
 (b) What is its maximum distance from the ground during its flight?

a) From equation

$$V^2 = V_0^2 + 2g(y - y_0) \Rightarrow$$

$$(18.8)^2 = V_0^2 + 2 \times 9.8 \times (18) \Rightarrow$$

$$(18.8)^2 = V_0^2 = (18.8)^2 - 2 \times 9.8 \times (18) = 0.64 \Rightarrow V_0 = 0.8 \text{ m/s}$$

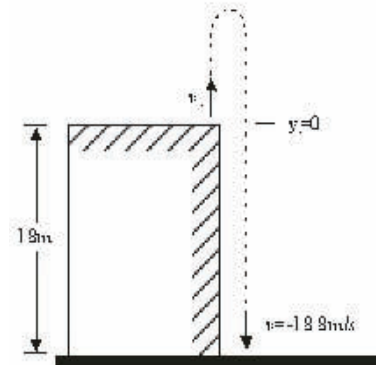


Figure (6.22)

(b) The maximum height reached by the stone is

$$h = \frac{V^2}{2g} = \frac{(18.8)^2}{2 \times 9.8} = 18 \text{ m}$$

Example (6.14)

A student throws a set of keys vertically upward to another student in a window 4m above as shown in Figure (6.23). The keys are caught 1.5s later by the student.

- (a) With what initial velocity were the keys thrown?
 (b) What was the velocity of the keys just before they were caught?

Solution**(a) we find**

$$y = y_0 + V_0 t - \frac{1}{2} g t^2$$

$$4 = 0 + 1.5 V_0 - \frac{1}{2} \times 9.8 \times (1.5)^2 \Rightarrow V_0 = 10.02 \text{ m/s}$$

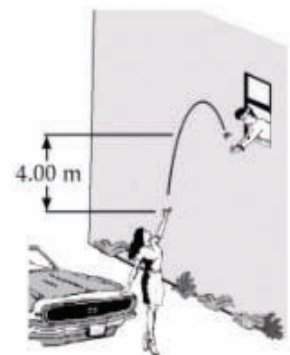


Figure (6.23)

(b) The velocity at any time $t > 0$ is given by

$$V = V_0 + at \Rightarrow$$

$$V = V_0 - gt \Rightarrow V = 10.02 - 9.8 \times 1.5 = -4.68 \text{ m/s}$$

6.6. Dynamics

is a branch of physics (specifically classical mechanics) concerned with the study of forces and torques and their effect on motion, as opposed to kinematics, which studies the motion of objects without reference to its causes.

There are many types of power found in nature, which are either mechanical or attractive or magnetic or electrical or nuclear. Figure (6.24)

We will examine this decision of the type I and II. Mechanical Power study we will begin to study Newton's laws of motion.

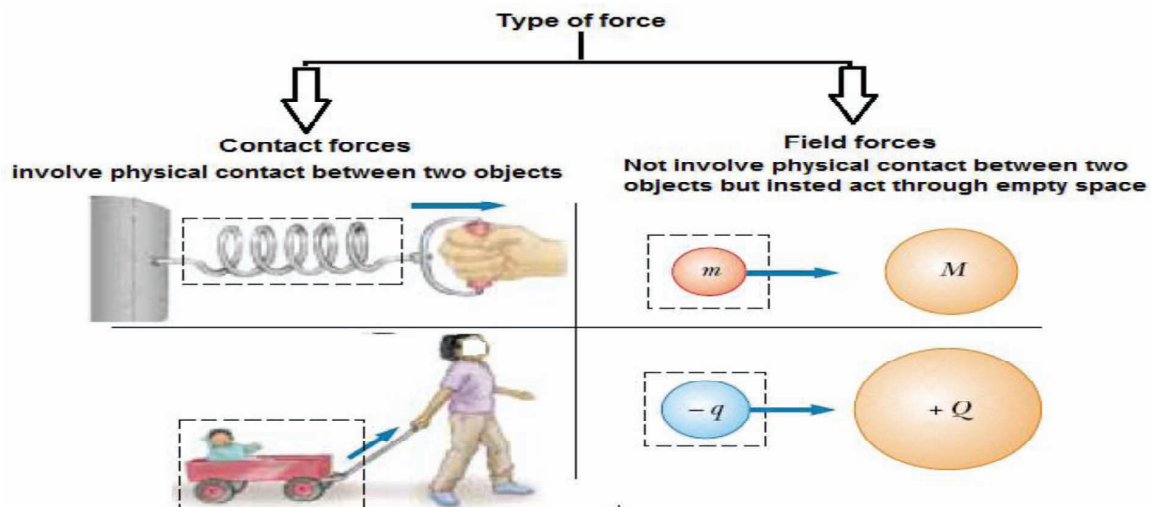


Figure (6.24)

6.6.1.The Concept of Force:

The force defined as: this exerted force can cause a change in velocity, and causes a body to accelerate. See figure (6.25)

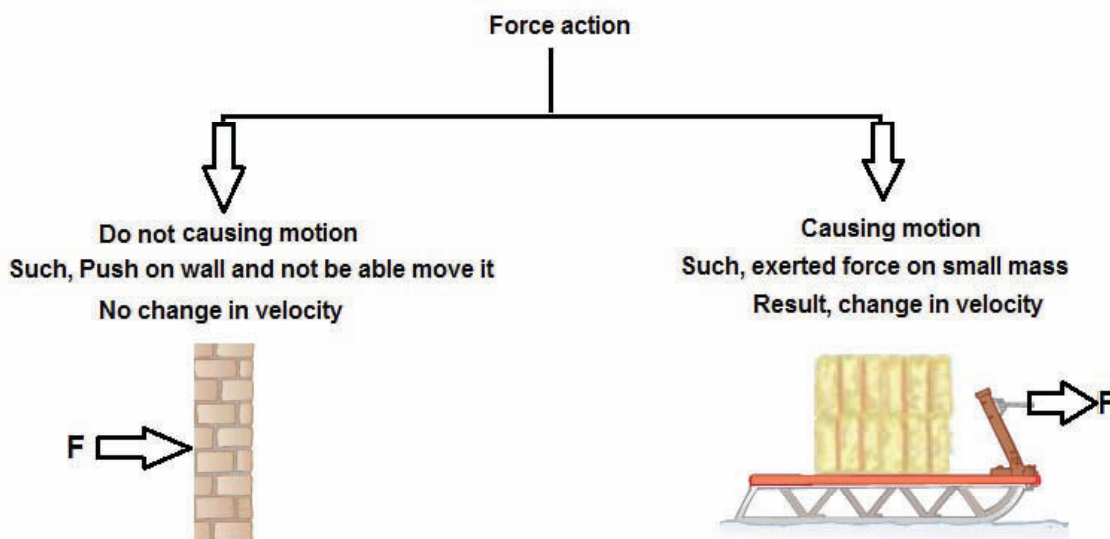


Figure (6.25)

6.6.1.1Normal Force (Perpendicular Force) “n”

is defined as this force prevents the object from falling through the surface and can have any magnitude needed to balance the downward force F_g

Also, Normal force (force perpendicular to motion)

Question) Why does the TV not accelerate in the direction of F_g at rest on a table, as shown in Figure (6.26)?

Answer:

The TV does not accelerate because the table exerts on the TV an upward force called the normal force.

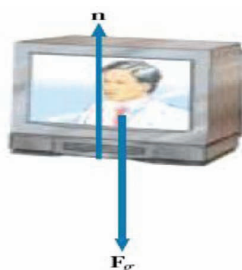


Figure (6.26)

$$m a = m g - n = 0 \quad (6.13)$$

Also, **The normal force** is a contact force that acts at the surface between two objects i.e. is just large enough to prevent objects from penetrating through each other and is not necessarily equal to the force of gravity in all situations.

Notes

If someone stacks books on the TV, the normal force exerted by the table on the TV increases.

Net Force

Because forces are vectors, we must add them as vectors

$$\vec{F}_{net} = \sum_i^n \vec{F}_i = \vec{F}_1 + \vec{F}_2 + \dots + \vec{F}_n$$

6.6.2 Newton's First Law (Force and Inertia):

The law of equilibrium states that an object at rest will remain at rest and an object in motion will remain in motion with a constant velocity unless acted on by a net external force

$$\vec{F}_{net} = \sum_i^n \vec{F}_i = 0 \quad (6.14)$$

$$\sum_i^n \vec{F}_{i,x} = 0 \quad \& \quad \sum_i^n \vec{F}_{i,y} = 0 \quad \& \quad \sum_i^n \vec{F}_{i,z} = 0$$

6.6.3. Newton's Second Law:

The law of acceleration, states that the acceleration of an object is directly proportional to the net force acting on it and inversely proportional to its mass.

$$a \propto F \Rightarrow F = ma$$

$$F = \frac{dp}{dt} = m \frac{dV}{dt} = ma \quad \text{or} \quad \sum \vec{F} = m\vec{a} \quad (6.15)$$

Units the force where m is the mass of the body by Kg and a is the acceleration of the body by m/s^2 , then the unit of the force is $(Kg.m/s^2)$ which is called Newton (N)

6.6.4. Newton's Third Law (Action and Reaction)

State that: The action force is equal in magnitude to the reaction force and opposite in direction,

If two objects interact, the force F_{12} exerted by object 1 on object 2 is equal in magnitude to and opposite in direction to the force F_{21} exerted by object 2 on object 1: See Figure (6.27)

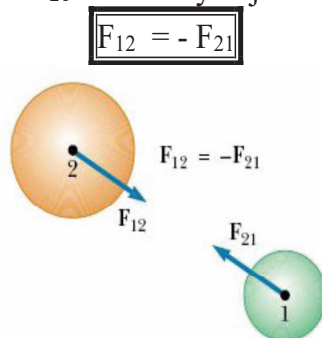


Figure (6.27)

- The force F_{12} is called **action force**
- the force F_{21} is called **reaction force**

Example (6.15):

Find the force required to move an object of mass 3000 Kg at an acceleration 3 m/s^2

Solution:

According to Newton's second law

$$F = m a \quad \text{then} \quad F = 3000(3) = 9000 \text{ N}$$

Example (6.16):

A small boat with an engine produces a force of 30000 N moving at acceleration 30 m/s^2 find its mass.

Solution:

According to Newton's second law

$$F = m a \Rightarrow m = F / a \Rightarrow m = 30000 / 30 \Rightarrow m = 1000 \text{ Kg}$$

Example (6.17):

Find the tension in a rope hanging a mass 100 Kg to the ceiling, where $g=9.8 \text{ m/s}^2$

Solution:

According to Newton's second law

$$T = mg \Rightarrow T = 100 (9.8) = 980 \text{ kg.m/s}^2$$

Example (6.18):

Two forces, F_1 and F_2 , act on a 5kg mass. If $F_1=20 \text{ N}$ and $F_2=15 \text{ N}$, find the acceleration in (a) and (b) of the Figure (6.28)



Figure (6.28)

Solution**The acceleration in Figure (6.27) (a)**

According to 2nd Newton law

$$\sum F = F_x i + F_y j = ma \Rightarrow \sum F = (20i + 15j) = 5a \Rightarrow$$

$$a = (4i + 3j) \text{ or } a = \sqrt{4^2 + 3^2} = \sqrt{25} = 5 \text{ m/s}^2$$

The acceleration in Figure (6.27) (b)

According to 2nd Newton law

$$\sum F = F_x i + F_y j = ma$$

$$F_{2x} = 15 \cos(60) = 7.5 \text{ N}$$

$$F_x = 20 \text{ N} + 7.5 \text{ N} = 27.5 \text{ N}$$

$$F_y = F_{2y} = 15 \sin(60) = 13 \text{ N}$$

$$\sum F = F_x i + F_y j = (27.5i + 13j) = ma = 5a$$

$$a = (5.5i + 2.6j) \text{ or } a = \sqrt{5.5^2 + 2.6^2} = \sqrt{37.01} = 6.084 \text{ m/s}^2$$

Example (6.19)

As shown in the figure, a 3kg block is pulled up a 35° inclined plane by a 40 N force parallel to the plane see Figure (6.29)

Calculate the acceleration of the block.

If it starts from rest, position after 2 second and normal force?

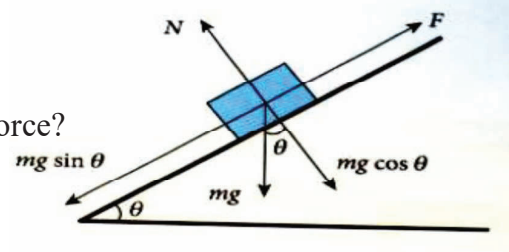


Figure (6.29)

Solution

We consider the inclined plane as the x-axis. The net force acting on the body along the x-axis is, considering the positive direction is the motion direction (up the plane)

$$\sum F_x = 0 \Rightarrow \text{then } F - mg \sin(\theta) = ma \Rightarrow$$

$$a = \frac{F - mg \sin(\theta)}{m} = \frac{40 - (3 \times 9.8 \times \sin 35)}{3} = 7.71 \text{ m/s}^2$$

Using the equation of motion of the form with the fact that the block starts from

$$x = v_0 t + \frac{1}{2} a t^2 \Rightarrow \text{but } v_0 = 0 \text{ (at rest) then } x = \frac{1}{2} a t^2$$

We find that the distance covered in 2 seconds is

$$x = \frac{1}{2} a t^2 = \frac{1}{2} \times 7.71 \times 2^2 = 15.42 \text{ m}$$

Normal force (force perpendicular to motion) is

$$\sum F_y = 0 \Rightarrow \text{then } N - mg \cos(\theta) = 0 \Rightarrow$$

$$N = mg \cos(\theta) = 3 \times 9.8 \times \cos(35) = 24.08 \text{ N}$$

Example (6.20)

A constant force of magnitude of 20N is applied, at angle of $\theta = 30^\circ$ above the horizontal to move a body of mass 4kg on a frictionless table as figure (6.30)

. Find the acceleration and the normal force on the body.

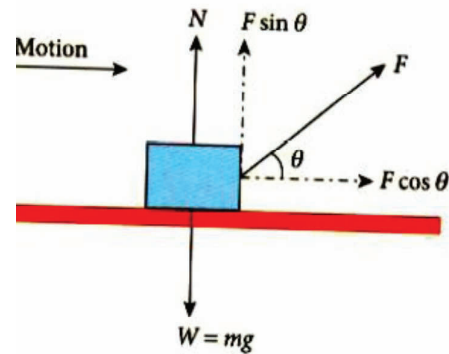


figure (6.30)

Solution

According to 2nd Newton law $\sum F = ma$

but motion in x-direction then

$$\sum F_x = ma \Rightarrow F \cos(\theta) = ma \Rightarrow$$

$$a = \frac{F \cos(\theta)}{m} = \frac{20 \times \cos 30}{4} = 4.33 \text{ m/s}^2$$

To find normal force on the box, we consider the motion along the y-direction with no acceleration, which is i.e

$$\sum F_y = 0 \Rightarrow N + F \sin(\theta) - mg = 0 \Rightarrow$$

$$N = mg - F \sin(\theta) = 4 \times 9.8 - 20 \sin 30 = 29.2 \text{ N}$$

6.7. Problems**Choose The Correct Answer in Each of The Followings:**

- 1) A particle moves along the x axis from x_i to x_f . Of the following values of the initial and final coordinates, which results in the displacement with the largest magnitude?
A) $x_i = 4\text{m}, x_f = 6\text{m}$
B) $x_i = -4\text{m}, x_f = -8\text{m}$
C) $x_i = -4\text{m}, x_f = 2\text{m}$
D) $x_i = -4\text{m}, x_f = 4\text{m}$

- 2) A car starts from Hither, goes 50 km in a straight line to Yon, immediately turns around, and returns to Hither. The time for this round trip is 2 hours.
The magnitude of the average velocity of the car (km/hour) in for this round trip is:
A) 0
B) 50
C) 100
D) 200

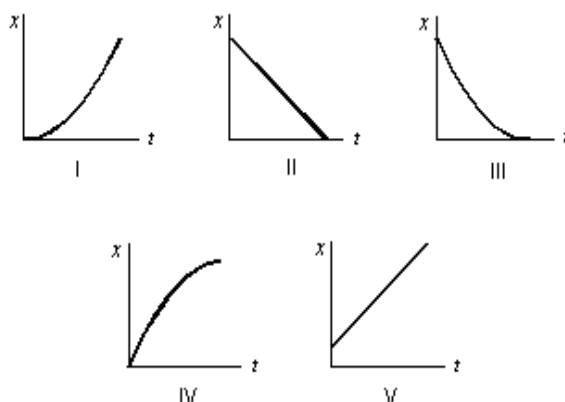
- 3) The coordinate of an object is given as a function of time by $x = 7t - 3t^2$, where x is in meters and t is in seconds.
Its average velocity over the interval from $t = 0$ to $t = 2$ s is:
A) 5 m/s
B) -5 m/s
C) 11 m/s
D) 1 m/s

- 4) The position y of a particle moving along the y axis depends on the time t according to the equation $y = at - bt^2$. The dimensions of the quantities a and b are respectively:
A) $\text{L}^2/\text{T}, \text{L}^3/\text{T}^2$
B) $\text{L}/\text{T}^2, \text{L}^2/\text{T}$
C) $\text{L}/\text{T}, \text{L}/\text{T}^2$
D) $\text{L}^3/\text{T}, \text{T}^2/\text{L}$

- 5) Two automobiles are 150 kilometers apart and traveling toward each other. One automobile is moving at 60 km/h and the other is moving at 40 km/h. In how many hours will they meet?
A) 2.5 h
B) 2.0 h
C) 1.75 h
D) 1.5 h

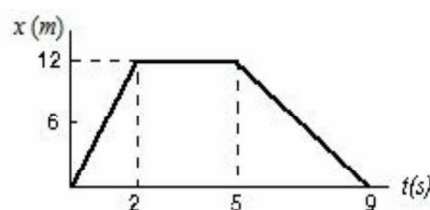
- 6) The coordinate of an object is given as a function of time by $x = 7t - 3t^2$, where x is in meters and t is in seconds. Its velocity at $t = 3$ s is:
A) -6 m/s
B) -11 m/s
C) -21 m/s
D) 9 m/s

7) Which of the following five coordinate versus time graphs represents the motion of an object whose speed is increasing?



- A) I
- B) II
- C) III
- D) IV and V

8) This graph shows the position of a particle as a function of time. What is its average velocity between $t = 5\text{ s}$ and $t = 9\text{ s}$?



- A) 3 m/s
- B) -3 m/s
- C) 12 m/s
- D) -12 m/s

9) Over a short interval near time $t = 0$ the coordinate of an automobile in meters is given by $x(t) = 27t - 4.0t^3$, where t is in seconds. At the end of 1.0 s the acceleration of the automobile is:

- A) 23 m/s^2
- B) 15 m/s^2
- C) -4.0 m/s^2
- D) -24 m/s^2

10) Over a short interval, starting at time $t = 0$, the coordinate of an automobile in meters is given by $x(t) = 27t - 4.0t^3$, where t is in seconds. The magnitudes of the initial (at $t = 0$) velocity and acceleration of the auto respectively are:

- A) 0 m/s; 12 m/s^2
- B) 0 m/s; 24 m/s^2
- C) 27 m/s; 0 m/s^2
- D) 27 m/s; 12 m/s^2

11) Starting at time $t = 0$, an object moves along a straight line with velocity in m/s given by $V(t) = 98 - 2t^2$, where t is in seconds. When it momentarily stops its acceleration is:

- A) 0 m/s^2
- B) -4.0 m/s^2
- C) -9.8 m/s^2
- D) -28 m/s^2

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